Pricing and Hedging without Fast Fourier Transform

Revisited, reliable and stable Fourier Transform Method for Affine Jump Diffusion Models
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Syllabus of the presentation

• Review of Fourier Methods in Option Pricing
• Calibration and Performance
• Greek Behavior of New FT-Q

Review of Fourier Methods in Option Pricing – theory

European Call Maturity Terminal Spot Price $S_T$
In AJD models Call Price can be expressed in a form close to the canonical Black-Scholes-Merton style

\[ C = S V_0 - K e^{-rT} V_T \]

where

\[ V_T = \Phi (\delta) \Phi (\delta_S) \cdot \ln [K] \]

determined by using the Levy’s inversion formula, i.e.:

\[ \phi_{\delta_S} = \frac{1}{2} \cdot \int_{-\infty}^{\infty} \left[ e^{i\phi} - 1 \right] d\delta \]

Review of Fourier Methods in Option Pricing – theory

requires a close formula for the Characteristic Function of the log-terminal price, i.e.:

\[ \hat{f}(\phi) = \int_{-\infty}^{\infty} e^{i\phi t} f(t) dt \]

has a closed formula for AJD models

How to compute:

Quadrature Algorithm
FT-Q for Fast Fourier Transform

Old FT - Q
New FT - Q

Review of Fourier Methods in Option Pricing – practice

Algorithms Valuation Criteria

STABILITY
The algorithm is defined stable if and only if

all settings of the quadrature scheme

"reasonable" result different from a NaN value

ACCURACY
The algorithm is defined accurate if and only if

pricing up to 10^-3 precision

SPEED
The algorithm is defined fast with respect to

the results of the FFT algorithm

a set of 400 prices along the strike

Pros (+)
STABILITY
SPEED

Cons (-)
ACCURACY

In order to overcome the cited problems of Old FT - Q:

Gauss - Lobatto Quadrature Algorithm
Re-adjustment of parameters

Cooley - Tukey algorithm
Applied to the equivalent formula via a recombinant FFT parameters

for ATM a

\[ \ln \left( \frac{S}{K} \right) \]

\[ \int_{-\infty}^{\infty} \left[ e^{i\phi t} - 1 \right] d\phi \]

\[ \phi_{\delta_S} = \frac{1}{2} \cdot \int_{-\infty}^{\infty} \left[ e^{i\phi t} - 1 \right] d\phi \]

\[ C = \int_{-\infty}^{\infty} \left[ e^{i\phi t} - 1 \right] d\phi \]
Review of Fourier Methods in Option Pricing - practice

Syllabus of the presentation

- Review of Fourier Methods in Option Pricing
- Calibration Procedure and Performance
- Greek Behaviour of New FT-Q

The Calibration Procedure and Performance

Pros (+)  Cons (-)

SPEED  STABILITY

Fast Fourier Transform

Pros (+)  Cons (-)

STABILITY  ACCURACY

FFT

Pros (+)  Cons (-)

STABILITY  ACCURACY

Accuracy (up to 20 times the quadrature algorithms)

The formula must be changed arbitrarily according to Option moneyness

The recombinant FFT parameters must be changed according to the choice of the pricing models

The Calibration Procedure and Performance

Pros (+)  Cons (-)

STABILITY  SPEED

Accuracy

By keeping in mind that only New FT-Q is stable and accurate, some figures on speed

Original Option Pricing Formulas are used

By now, the speed of Fourier Transform method is closer than ever to the FFT calibration time

Calibration Performances using Option Readjusted Pricing Formulas

Where available

Greek behaviour of new FT-Q

An impressive methodology to test Stability of the New FT - Quadrature algorithm is to compute Greeks

Indeed, in an AJD setting the Greeks are available in closed form

So, an extended spanning of the AJD Greeks on the parameters set is useful to assess models and test Stability

Calibration Performances using Option Readjusted Pricing Formulas

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<td>18.32 sec.</td>
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<td>BCC Model</td>
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