CONSOB Commissione Nazionale per le Società e la Borsa

A quantitative approach to detect market abuses:

The surveillance automatic integrated system (SAIS)

Market Abuses

- Insider Trading
- Manipulation
  - Market Based Manipulation
  - Information Based Manipulation

The problem for the Supervisory Authorities is:

The real time identification of market abuse phenomena

Market Abuse Detection

The real time detection of market abuse phenomena requires as a first step for each stock the determination on daily basis of signals of abnormalities

(so-called **FAILURES**)

...that's because...

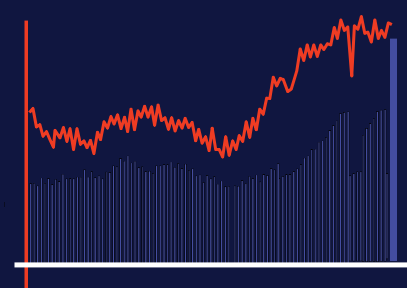
Actions which may be attributed to market abuse phenomena

Failure

Particular occurrence/event referred to a specific stock

How to detect a failure?

Through the exam of the elementary components which mainly affect the pattern of a stock and which characterise the trades made by the intermediaries



The elementary components:

How to examine the elementary components in order to detect a failure?

The Financial Literature

The Supervisory Experience

Quantitative Models

Prices

The Financial Literature

- The trading prices have to be analysed in terms of returns, through the study of the dynamics of the logarithm of the price;
- Auto-regressive models in discrete time capture both the mean reversion and the momentum effect components of the returns;

The Supervisory Experience

- Stock returns generally undergo sharp changes (for example at moment insider information is disclosed) or show movements that cannot be attributed to a mean-reverting type dynamic (for example in the presence of manipulation);

- The presence of abnormal returns is disclosed through an estimation of the returns which may be realysed employing diffusive processes

Quantities

The Financial Literature and the Supervisory Experience

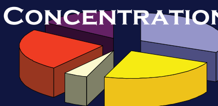
- The quantities traded by each intermediary are examined in an aggregate way in terms of daily trading volumes according to an auto-gressive scheme
- The market composition is assessed through two levels of analysis:
  - the level of concentration of the intermediaries, that is the number of intermediaries and their shares in terms of trading volumes (so-called static concentration);
  - the evolution of the concentration of the intermediaries, that is the change of each intermediary's share in terms of trading volumes on a given security (so-called dynamic concentration).

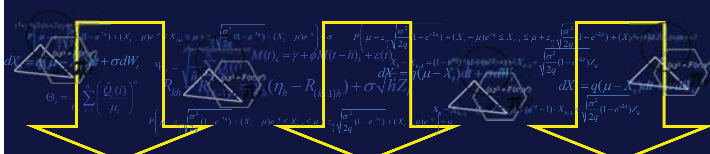
...hence, a Market Abuse Detection Procedure ...

...requires the control of 4 financial variables:

- Prices
- Volumes
- Static Concentration
- Dynamic Concentration

Static and Dynamic Concentration





...in order to implement models with predictive capabilities which allow the identification of abnormal movements in the variable examined (so-called Alerts)

Financial Variable ⟷ Reference Model

Alert

The joint reading of the results of the various alerts identifies the stocks for which there is a failure, which becomes the Warning for the Consob

The S.A.I.Vi.M: the procedure for the Market Abuse Detection - implementation

Alerts — Construction of the tripwires in order to detect abnormal movements of the financial variables: so-called Alert

Warning — Definition of the algorithm that - by reading the alerts – identifies on a daily basis the stocks which entails some Failures, so-called Warning

Supervisory Activity — Comprehension of the reasons underlying the Warning and consequent decisions

The S.A.I.Vi.M: functioning

For each stock listed on the Market

< 3 Alert → No → End of the procedure

≥ 3 Alert → Yes → Enforcement Actions

Yes ↓ Tools of 2nd level

Tools of 2nd level → No → End of the procedure

Tools of 2nd level → Yes → Enforcement Actions

Construction of the S.A.I.Vi.M: Main Problems

- The stocks listed on the Market are different as regards:
  - Liquidity
  - Sector to which they belong
  - P/E
  - ….
- The market is characterised by moments of boost/ "euphory" or of "crisis" which may be generalized or bounded to some sectors (for example , the 2000 bubble on Technology stocks)
- The time horizon for the Failures analysis cannot be too long (for instance: a quarter, a semester , a year) in order to avoid the risk of losing sensitivity:
  - Changes in the Strategic Area of Business of the company;
  - New listings
  - …
- The construction of the tripwires and of the algorithm which produces the warning needs to be valid over all the stocks and to preserve the adequacy of its performance over time

Il S.A.I.Vi.M: The choice of the Models

- Developing the models for the tripwires through **the employment of diffusive processes:** that's because diffusive processes exploiting some results of the stochastic limit theory prove to be:
  - Extremely suitable/proper for the representation of the phenomena
  - Good-performing even when the number of the observations is low
  - Able to simplify the problems concerning the estimation and the stability of the parameters

FINANCIAL VARIABLE

DIFFUSIVE PROCESSES
(STOCHASTIC DIFFERENTIAL EQUATIONS)

ALERT

THE S.A.I.VI.M.: THE CALIBRATION OF THE PROCEDURE

THE REFERENCE SAMPLE

THE SET OF STOCKS AND OF THE RELATIVE OBSERVATION PERIODS HAS BEEN SELECTED BY LOOKING AT THOSE CASES FOR WHICH BOTH THE FAILURES AND THEIR REASONS WERE KNOWN

THE STOCKS SELECTION (N.26) WAS ORIENTED BY:

- THE PRESENCE OF AN INVESTIGATION CARRIED ON BY CONSOB;
- THE EXISTENCE OF A CONSOB SIGNALLING TO THE JUDICIAL AUTHORITY REGARDING AN HYPOTHESIS OF MARKET ABUSE;
- THE LIQUIDITY OF THE STOCK;
- THE HISTORICAL VOLATILITY OF THE STOCK;
- THE *PRICE/EARNING RATIO* OF THE STOCK;
- THE DIFFUSION/SPREADING OF THE STOCK ON THE MARKET.

THE S.A.I.VI.M.: THE CALIBRATION OF THE PROCEDURE

THE REFERENCE SAMPLE

THE SET OF STOCKS AND OF THE RELATIVE OBSERVATION PERIODS HAS BEEN SELECTED BY LOOKING AT THOSE CASES FOR WHICH BOTH THE FAILURES AND THEIR REASONS WERE KNOWN

THE SELECTION OF THE TIME HORIZONS (AVERAGE=20 MONTHS) WAS ORIENTED BY:

- THE PERIOD OF THE INVESTIGATION
- THE MOMENT IN WHICH THE STOCK WAS LISTED
- THE OPERATIONS OF M&A
- THE MOMENT OF THE STOCK *DE-LISTING*

THE S.A.I.VI.M.: THE CALIBRATION OF THE PROCEDURE

THE REFERENCE SAMPLE

AIM: TO EXPLAIN THE FAILURES OBSERVED IN THE REFERENCE SAMPLE

(PROBLEM OF STOCHASTIC PROGRAMMING)

ALERTS

WARNING

CHOICE OF THE DISCRETE PROCESS, DERIVATION OF THE RELATIVE DIFFUSIVE PROCESS AND ESTIMATION OF THE PARAMETERS FOR EACH FINANCIAL VARIABLE

CHOICE OF THE ALGORITHM FOR THE IDENTIFICATION OF THE FAILURE ON THE STOCK, I.E. THE CONSOB WARNING

THE S.A.I.VI.M. AND THE EMPIRICAL EVIDENCE: MAIN RESULTS

- ALL THE TRADING PERIODS HIGHLIGHTED AS CRITICAL IN THE REPORTS FOR THE COMMISSION, SINCE RELATED TO MARKET ABUSE PHENOMENA, HAVE BEEN DETECTED
- MOREOVER HAVE BEEN HIGHLIGHTENED OTHER PERIODS CHARACTERISED BY ONE OF THE FOLLOWING SITUATIONS:
  - THE PRESENCE OF *RUMORS* ON THE MARKET, THAT IS OF NEWS HAVING THE POTENTIAL TO BE *PRICE SENSITIVE*;
  - THE EXISTENCE OF CONSIDERABLE CHANGES IN THE MOVEMENTS OF THE FINANCIAL VARIABLES ANALYSED.

EMPIRICAL EVIDENCE: SOME NUMBERS

N° OF DAYS EXAMINED	N° OF WARNINGS
10.193	267

EMPIRICAL EVIDENCE: SOME NUMBERS

INFORMATIONAL REFERENCE OF THE WARNING	%
REPORT TO THE COMMISSION	22%
CONSOB NEWS	11%
BALANCE SHEET	10%
INFORMATION ON THE NET	53%
DATA ANALYSIS	4%

CONSTRUCTION OF THE ALERTS

- STATIC CONCENTRATION
- DYNAMIC CONCENTRATION

- RETURNS
- VOLUMES

CONSTRUCTION OF THE INDICATOR

THE PRICE ALERT

6 LOGICAL AND COMPUTATIONAL STEPS

EMPLOYMENT OF AN AUTO-REGRESSIVE MODEL APPLIED TO THE LOGARITHMIC TRANSFORMATION OF THE PRICES

THE LOGARITHMIC TRANSFORMATION

$$R_t = log P_t$$

1) I. THE PROCESS IN DISCRETE TIME: AR(1)

$$R_k = \alpha + \lambda R_{k-1} + \hat{\sigma} Z_k$$

$Z_k \sim N(0,1)$

$R_k = Log(P_k)$

P_k is the stock price observed at time k

1) II. THE AR(1) PROCESS IN DIFFERENTIAL TERMS

by defining λ=1-γ e $\alpha = \gamma \cdot \eta$

$$R_k - R_{k-1} = \gamma \cdot (\eta - R_{k-1}) + \hat{\sigma} Z_k$$

2) THE RE-SCALING OF THE PROCESS:

THE k INTERVALS ARE DIVIDED INTO 1/h SUBINTERVALS WITH A LENGTH h

$$R_{kh}-R_{(k-1)h}=\gamma_h(\eta_h-R_{(k-1)h})+\sigma\sqrt{h}Z_k$$

OR

$$R_{kh}-R_{(k-1)h}=\gamma_h(\eta_h-R_{(k-1)h})+\sigma Z_{kh}$$

$Z_{kh}\sim N(0,\sqrt{h})$

3) THE WEAK CONVERGENCE FOR $h\downarrow 0$

$$R_{kh}-R_{(k-1)h}=\gamma_h(\eta_h-R_{(k-1)h})+\sigma R_{kh}$$

LIM $h\downarrow 0$

$$dR_t=q(\mu-R_t)dt+\sigma dW_t$$

4) THE SDE PROPERTIES

(ORNSTEIN-UHLENBECK ARITHMETIC PROCESS)

$$R_t\sim N\left((R_{t-1}-\mu)e^{-q}+\mu;\sqrt{\frac{\sigma^2}{2q}(1-e^{-2q})}\right)$$

5) I. THE RELATION DISCRETE VERSUS CONTINUOUS AND THE ESTIMATION OF THE PARAMETERS

THE SPECIFICATION OF THE AR (1) PROCESS ALLOWS TO AVOID USELESS NUMERICAL PROCEDURES

$$R_k-R_{k-1}=\gamma(\eta-R_{k-1})+\hat{\sigma}Z_k$$

EMPLOYMENT OF THE SDE PROPERTIES

$$dR_t=q(\mu-R_t)dt+\sigma dW_t$$

5) II. THE RELATION DISCRETE VERSUS CONTINUOUS TIME AND THE ESTIMATION OF THE PARAMETERS

(IMPOSING THE EQUALITY BETWEEN THE FIRST AND THE SECOND CONDITIONAL MOMENTS)

$$R_k-R_{k-1}=(1-e^{-q})\cdot\mu+(e^{-q}-1)\cdot R_{k-1}+\sqrt{\frac{\sigma^2}{2q}(1-e^{-2q})}Z_k$$

5) III. THE RELATION DISCRETE VERSUS CONTINUOUS TIME AND THE ESTIMATION OF THE PARAMETERS

$$R_k-R_{k-1}=(1-e^{-q})\cdot\mu+(e^{-q}-1)\cdot R_{k-1}+\sqrt{\frac{\sigma^2}{2q}(1-e^{-2q})}Z_k$$

REGRESSION ANALYSIS

$$R_k-R_{k-1}=\hat{a}+\hat{b}R_{k-1}+\varepsilon_k$$

5) IV. THE RELATION DISCRETE VERSUS CONTINUOUS TIMEAND THE ESTIMATION OF THE PARAMETERS

$$\mu = -\frac{a}{\hat{b}}$$

$$q = log(\hat{b}+1)^{-1}$$

$$\sigma = \sqrt{\sum_k \frac{\varepsilon_k^2}{n-2}} \cdot \sqrt{\frac{log(\hat{b}+1)^2}{\hat{b}^2 + 2\hat{b}}}$$

$k = 15$ ⇒ INFRA-MONTHLY ANALYSIS

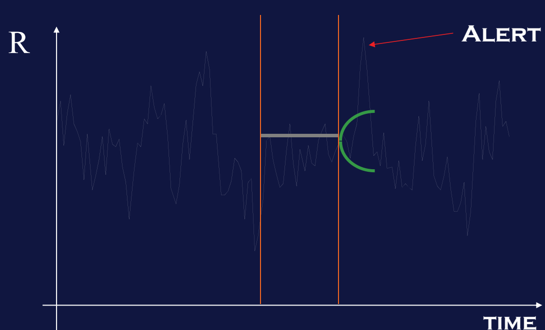
6) THE DETECTION OF THE ABNORMAL PATTERN FOR THE F.V:

THE NORMALITY PREDICTION INTERVAL

$$P\left(\begin{array}{c} \mu - z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma^2}{2q}(1-e^{-2q})} + (R_t - \mu)e^{-q} \leq \\ \leq R_{t+1} \leq \\ \leq \mu + z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma^2}{2q}(1-e^{-2q})} + (R_t - \mu)e^{-q} \end{array}\right) = \alpha$$

THE ALERT ACTIVATION

EXAMPLE: THE PRICE/RETURN ALERT



THE VOLUME ALERT

6 LOGICAL AND COMPUTATIONAL STEPS

RAW DATA EXAMINED ACCORDING TO AN AUTOCORRELATION SCHEME

THE RAW DATA

$$Q_t = \sum_i A(i) + V(i)$$

A = *purchases*
V = *sales*
j denotes the intermediary

THE MODELS IN DISCRETE AND IN CONTINUOUS TIME

$$Q_k - Q_{k-1} = -\gamma Q_{k-1} + \hat{\sigma} Z_k$$

$$dQ_t = -\theta Q_t dt + \sigma dW_t$$

THE SPECIFIED DISCRETE PROCESS AND THE PARAMETERS ESTIMATION

$$Q_k - Q_{k-1} = (e^{-\theta} - 1) \cdot Q_{k-1} + \sqrt{\frac{\sigma^2}{2\theta}(1 - e^{-2\theta})} Z_k$$

$$\theta = log(\hat{b}+1)^{-1}$$

$$\sigma = \sqrt{\sum_k \frac{\varepsilon_k^2}{n-1}} \cdot \sqrt{\frac{log(\hat{b}+1)^2}{\hat{b}^2 + 2\hat{b}}}$$

$k = 15$ ⇨ INFRA-MONTHLY ANALYSIS

THE NORMALITY PREDICTION INTERVAL

$$P\left(\begin{array}{c} z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma^2}{2\theta}(1 - e^{-2\theta})} + Q_t e^{-\theta} \le \\ \le Q_{t+1} \le \\ \le \mu + z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma^2}{2\theta}(1 - e^{-2\theta})} + Q_t e^{-\theta} \end{array} \right) = \alpha$$

THE ALERTS ON THE CONCENTRATION

⇩

DEFINITION OF A SYNTHETIC INDICATOR

⇩

DATA EXAMINED ACCORDING TO A AUTOCORRELATION SCHEME

SEE TECHNICAL NOTE

STATIC CONCENTRATION

ENTROPY INDEX

$$\Theta_t = \frac{1}{n_t} \sum_{i=1}^{n_t} \left(\frac{\hat{Q}_t(i)}{\mu_t} \right)^{\alpha}$$

WHERE

$$\hat{Q}_t(i) = \sum_{i=1}^{n_t} Q_{t-5}(i) \qquad \mu_t = \frac{\sum_{i=1}^{n_t} \hat{Q}_t(i)}{n_t}$$

n_t IS THE NUMBER OF INTERMEDIARIES PRESENT ON THE MARKET AT TIME t

$Q_t(i),\ i = 1, \ldots, n_t$ ARE THE QUANTITIES TRADED BY THE ith INTERMEDIARY AT TIME t

STATIC CONCENTRATION

CONSIDERATION/REMARK:

- THE NEED TO CAPTURE NOT ONLY THE MOVEMENT IN THE VARIABLE FOR THE TOTAL TURNOVER OF THE MARKET BUT ALSO THE POSSIBLE DIRECTIONS TAKEN BY INDIVIDUAL INTERMEDIARIES AND, HENCE THE MARKET, REQUIRES THE DEFINITION OF 3 DIFFERENTS PRE-ALERTS

STATIC CONCENTRATION

THE PRE-ALERTS

QUANTITIES BOUGHT

$$Q_t^A = \sum_i A(i)$$

GROSS TURNOVER

$$Q_t = \sum_i A(i) + V(i)$$

QUANTITIES SOLD

$$Q_t^V = \sum_i V(i)$$

STATIC CONCENTRATION

SEE THE TECHNICAL NOTE FOR THE COMPLETE DESCRIPTION OF THE MATHEMATICS ON:

- THE MODELS IN DISCRETE AND IN CONTINUOUS TIME
- THE SPECIFIED DISCRETE PROCESS AND THE PARAMETERS ESTIMATION
- THE NORMALITY PREDICTION INTERVAL

STATIC CONCENTRATION

THE ALERT'S GENERATION

Θ_t^A

Θ_t^V

Θ_t

AT LEAST ONE OF THE THREE PRE-ALERTS PRODUCES 1 ALERT

ALERT

STATIC CONCENTRATION

CONSIDERATION/REMARK:

- THROUGH SOME EASY MATHEMATICAL STEPS/PASSAGES IT IS POSSIBLE TO IDENTIFY THE INTERMEDIARIES WHO GENERATED THE ALERT

DYNAMIC CONCENTRATION

DISSIMILARITY INDEX

$$\Psi_t = \sqrt{\frac{1}{\tilde{n}_t}\sum_{i=1}^{\tilde{n}_t}\tilde{Q}_t(i)^2}$$

WHERE

$$\tilde{Q}_t(i) = Q_t(i) - Q_{t-k}(i)$$

$$\tilde{n}_t \doteq n_t : \tilde{Q}_t(i) \neq 0$$

DYNAMIC CONCENTRATION

CONSIDERATION/REMARK:

- THE NEED TO CAPTURE NOT ONLY THE MOVEMENT IN THE VARIABLE FOR THE TOTAL TURNOVER OF THE MARKET BUT ALSO THE POSSIBLE DIRECTIONS TAKEN BY INDIVIDUAL INTERMEDIARIES AND, HENCE THE MARKET, REQUIRES THE DEFINITION OF 3 DIFFERENTS PRE-ALERTS

DYNAMIC CONCENTRATION

THE PRE-ALERTS

QUANTITIES BOUGHT

$$Q_t^A = \sum_i A(i)$$

NET TURNOVER

$$Q_t = \sum_i A(i) - V(i)$$

QUANTITIES SOLD

$$Q_t^V = \sum_i V(i)$$

DYNAMIC CONCENTRATION

SEE THE TECHNICAL NOTE FOR THE COMPLETE DESCRIPTION OF THE MATHEMATICS ON:

- THE MODELS IN DISCRETE AND IN CONTINUOUS TIME
- THE SPECIFIED DISCRETE PROCESS AND THE PARAMETERS ESTIMATION
- THE NORMALITY PREDICTION INTERVAL

DYNAMIC CONCENTRATION

THE ALERT'S GENERATION

Ψ_t^A

Ψ_t^V

Ψ_t

AT LEAST ONE OF THE THREE PRE-ALERTS PRODUCES 1 ALERT

ALERT

DYNAMIC CONCENTRATION

CONSIDERATION/REMARK:

- THROUGH SOME EASY MATHEMATICAL STEPS/PASSAGES IT IS POSSIBLE TO IDENTIFY/SPOT THE INTERMEDIARIES WHO GENERATED THE ALERT

CONSTRUCTION OF THE ALGORITHM FOR THE GENERATION OF THE WARNING

R_t

Q_t

Θ_t

Ψ_t

AT LEAST THREE OF THE FOUR ALERTS GENERATE THE WARNING

WARNING

THE SOFTWARE IMPLEMENTATION OF THIS PROCEDURE OF MARKET ABUSE DETECTION REPRESENTS:

CONSOB

COMMISSIONE NAZIONALE PER LE SOCIETA' E LA BORSA

AUTOMATIC INTEGRATED SYSTEM FOR MARKET SURVEILLANCE