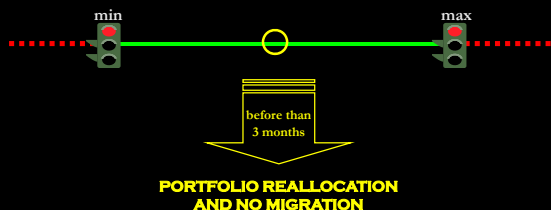
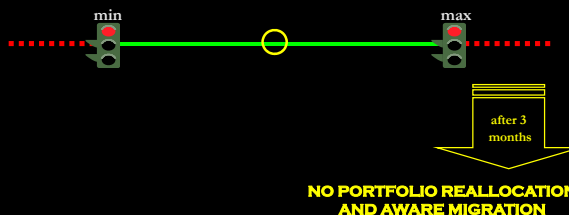


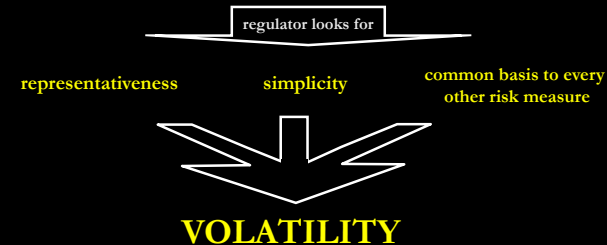
wide enough intervals do imply aware migration



wide enough intervals do imply aware migration



a bunch of risk metrics in finance: which one to choose?

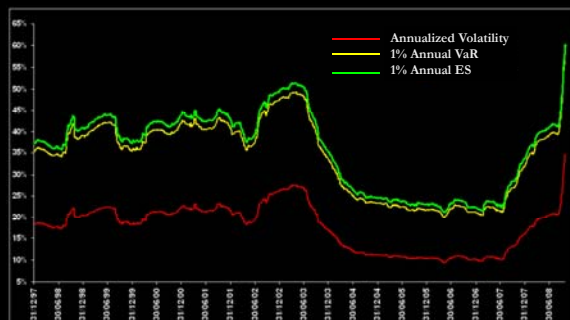


a bunch of volatility measures: which one to choose?



Annualized Volatility of Daily NAV Returns

1:1 correspondance with other risk measures



Syllabus

- The Risk Profile of Mutual Funds
 - Representation via Qualitative Risk Classes
 - Adequate Underlying Risk Metric
- Diffusive GARCH
 - Intuition
 - The Weak Convergence Theorem on R^2
 - The Continuous Limit of the M-GARCH(1,1)
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- The Grid of Volatility Intervals
 - Loss Intervals
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 - Intervals Fine-Tuning: the Iterative Procedure
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 - The Evolution of the Risk Profile over Time

• Conclusions

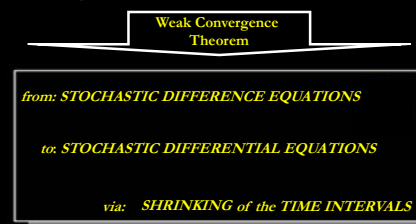
need for volatility forecasts

need for volatility forecasts

need for volatility forecasts

through the diffusion limit of GARCH

through the diffusion limit of GARCH



statement

The sequence $\{X_t^h\}$, whose measurable space is $(\mathbb{R}^2, \mathcal{B}(\mathbb{R}^2))$, converges weakly for $h \downarrow 0$ to the process $\{X_t\}$ which has a unique distribution and is characterized by the following stochastic differential equation:

$$dX_t = b(x, t)dt + \sigma(x, t)dW_{2,t}$$

where $W_{2,t}$ is a two-dimensional standard Brownian motion, if the conditions 1-4, presented below, are satisfied.

statement

The process $\{X_t\}$ has a distribution independent on the choice of $\sigma(x, t)$ and it takes finite values over finite time intervals, i.e. $\forall T > 0$:

$$P\left(\sup_{0 \leq t \leq T} \|X_t\| < \infty\right) = 1$$

conditions:

n. 1

If \exists a $\delta > 0$ s.t.:

$$\lim_{h \downarrow 0} \begin{pmatrix} c_{h,\delta}(x_1, t) \\ c_{h,\delta}(x_2, t) \end{pmatrix} = 0$$

conditions:

n. 1

If \exists a $\delta > 0$ s.t.:

$$\lim_{h \downarrow 0} \begin{pmatrix} c_{h,\delta}(x_1, t) \\ c_{h,\delta}(x_2, t) \end{pmatrix} = 0$$

then \exists

$$a(x, t) \lim_{h \downarrow 0} \begin{pmatrix} a_h(x_1, t) & a_h((x_1, x_2), t) \\ a_h((x_2, x_1), t) & a_h(x_2, t) \end{pmatrix} = \begin{pmatrix} a(x_1, t) & 0 \\ 0 & a(x_2, t) \end{pmatrix}$$

s.t.

$$b(x, t) \lim_{h \downarrow 0} \begin{pmatrix} b_h(x_1, t) \\ b_h(x_2, t) \end{pmatrix} = \begin{pmatrix} b(x_1, t) \\ b(x_2, t) \end{pmatrix}$$

conditions:

n. 2

$\exists \sigma(x, t)$ s.t.: $\forall x_1 \in \mathbb{R}^1, \forall x_2 \in \mathbb{R}^1$,

it holds

$$\begin{pmatrix} \sigma(x_1, t) & 0 \\ 0 & \sigma(x_2, t) \end{pmatrix} = \begin{pmatrix} \sqrt{a(x_1, t)} & 0 \\ 0 & \sqrt{a(x_2, t)} \end{pmatrix}$$

conditions:

n. 3

For $h \downarrow 0$, X_0^h converges in distribution to a random variable X_0 with probability measure ν_0 on $(\mathbb{R}^2, \mathcal{B}(\mathbb{R}^2))$

n. 4

ν_0 , $a(x, t)$ and $b(x, t)$ uniquely specify the distribution of the process $\{X_t\}$ characterized by an initial distribution ν_0 , a conditional second moment $a(x, t)$ and a conditional first moment $b(x, t)$

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statement

from the M-GARCH(1,1)

$$\begin{cases} X_k - X_{k-1} = \gamma \cdot (\eta - X_{k-1}) + \sigma_k \bar{Z}_k \\ \text{and} \\ \ln \sigma_{k+1}^2 - \ln \sigma_k^2 = \beta_0^{(k)} + (\beta_1^{(k)} - 1) \ln \sigma_k^2 + \beta_1^{(k)} \ln Z_k^2 \\ \text{or, equivalently} \\ \ln \sigma_{k+1}^2 - \ln \sigma_k^2 = \beta_0^{(k)} + (\beta_1^{(k)} - 1) \ln \sigma_k^2 + 2\beta_1^{(k)} \ln |Z_k| \end{cases}$$

\bar{Z}_k and Z_k are i.i.d. $N(0,1)$

statement

from the M-GARCH(1,1)

$$\begin{cases} X_k - X_{k-1} = \gamma \cdot (\eta - X_{k-1}) + \sigma_k \bar{Z}_k \\ \text{and} \\ \ln \sigma_{k+1}^2 - \ln \sigma_k^2 = \beta_0^{(k)} + (\beta_1^{(k)} - 1) \ln \sigma_k^2 + \beta_1^{(k)} \ln Z_k^2 \\ \text{or, equivalently} \\ \ln \sigma_{k+1}^2 - \ln \sigma_k^2 = \beta_0^{(k)} + (\beta_1^{(k)} - 1) \ln \sigma_k^2 + 2\beta_1^{(k)} \ln |Z_k| \end{cases}$$

\bar{Z}_k and Z_k are i.i.d. $N(0,1)$

Weak Convergence Theorem

$$dX_t = q(\mu - X_t)dt + \sigma_t dW_t$$

$$d \ln \sigma_t^2 = (\beta_0 + 2\beta_1 E(\ln |Z_t|) + (\beta_1 - 1) \ln \sigma_t^2) dt + 2|\beta_1| \sqrt{\text{Var}(\ln |Z_t|)} dW_t^*$$

Z_t is $N(0,1)$

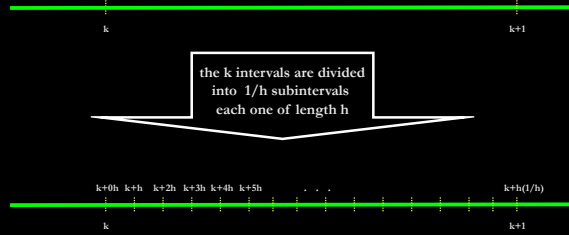
proof

step 1: rescaling of the discrete process



proof

step 1: rescaling of the discrete process



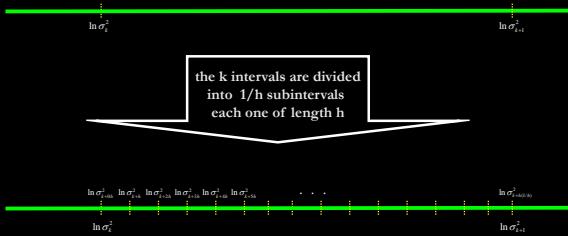
proof

step 1: rescaling of the discrete process



proof

step 1: rescaling of the discrete process



proof

step 1: rescaling of the discrete process

relationship from rescaling

$$\ln \sigma_k^2 - \ln \sigma_{k-1}^2 = \sum_{j=1}^{\frac{1}{h}} \left(\ln \sigma_{(k-1)+jh}^2 - \ln \sigma_{(k-1)+j(h-h)}^2 \right)$$

proof

step 1: rescaling of the discrete process

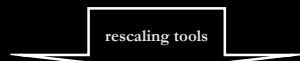
rescaling tools

$$(\ln |Z|)_{kh} = \sqrt{h} \ln |Z_{k-1}| + (h - \sqrt{h}) E(\ln |Z_{k-1}|)$$

where: $f_{Z_{kh}}(z_{kh}) = (1/\sqrt{h}) f_{Z_k}(z_{kh})$

proof

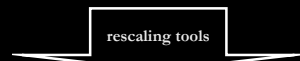
step 1: rescaling of the discrete process



$$\beta_{0h} = \beta_0^{(k)} \cdot h$$

proof

step 1: rescaling of the discrete process

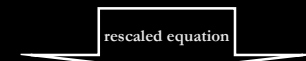


$$\beta_{1h} \text{ s.t. :}$$

$$0 = (\beta_0 + \beta_1 \ln \sigma_{k-1}^2 + 2\beta_1 \ln |Z_{k-1}|) - \beta_{1h}^{\frac{1}{h}} \ln \sigma_{k-1}^2 - (\beta_0 h + 2\beta_{1h} (\sqrt{h} \ln |Z_{k-1}| + (h - \sqrt{h}) E(\ln |Z_{k-1}|))) \sum_{j=1}^{\frac{1}{h}} \beta_{1h}^j$$

proof

step 1: rescaling of the discrete process



$$\ln \sigma_{kh}^2 = \beta_{0h} + \beta_{1h} \ln \sigma_{(k-1)h}^2 + 2\beta_{1h} (\sqrt{h} \ln |Z_{k-1}| + (h - \sqrt{h}) E(\ln |Z_{k-1}|))$$

proof

step 2: construction of the process $\{\ln \sigma_t^{2h}\}$

definition of the prob. measure P_h on the Skorokhod space D s.t.:

$$P_h(\ln \sigma_0^{2h} \in \Gamma) = \nu_0(\Gamma) \quad \forall \Gamma \in \mathcal{B}(\mathbb{R}^1)$$

$$P_h(\ln \sigma_t^{2h} = \ln \sigma_{kh}^2, \quad \forall kh \leq t < (k+1)h) = 1$$

$$P_h(\ln \sigma_{(k+1)h}^2 \in \Gamma | \widehat{\mathcal{S}}_{kh}) = \Pi_{h,kh}(\ln \sigma_{kh}^2, \Gamma) \text{ a.s. under } P_h, \forall k \geq 0, \forall \Gamma \in \mathcal{B}(\mathbb{R}^1)$$

proof

step 3: check of condition n. 1 of the weak convergence th.

$$\beta_{0h} := \beta_0 \cdot h$$

$$\beta_{1h} := \beta_1 \cdot h$$

$$\lim_{h \downarrow 0} c_{h, \beta-1}(\widehat{\ln \sigma^2}, t) = 0$$

$$\lim_{h \downarrow 0} b_h(\widehat{\ln \sigma^2}, t) = (\beta_0 + 2\beta_1 \mathbf{E}(\ln |Z_t|) + (\beta_1 - 1) \ln \sigma_t^2)$$

$$\lim_{h \downarrow 0} a_h(\widehat{\ln \sigma^2}, t) = 4\beta_1^2 \text{Var}(\ln |Z_t|)$$

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proof

step 2: construction of the process $\{\ln \sigma_t^{2h}\}$

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$$P_h(\ln \sigma_{(k+1)h}^2 \in \Gamma | \widehat{\mathcal{S}}_{kh}) = \Pi_{h,kh}(\ln \sigma_{kh}^2, \Gamma) \text{ a.s. under } P_h, \forall k \geq 0, \forall \Gamma \in \mathcal{B}(\mathbb{R}^1)$$

$$\ln \sigma_{t+h}^{2h} - \ln \sigma_t^{2h} = \beta_{0h} + (\beta_{1h} - h) \ln \sigma_t^{2h} + 2\beta_{1h} \left\{ \sqrt{h} \ln |Z_t^h| + (h - \sqrt{h}) E(\ln |Z_t^h|) \right\}$$

proof

step 3: check of condition n. 1 of the weak convergence th.

Condition n. 2 is verified for every $\sigma > 0$, i.e.:

$$\sigma(\ln \widehat{\sigma^2}, t) = 2|\beta_1| \sqrt{\text{Var}(\ln |Z_t|)}$$

Condition n. 3 is satisfied by construction of the process $\{\ln \sigma_t^{2h}\}$

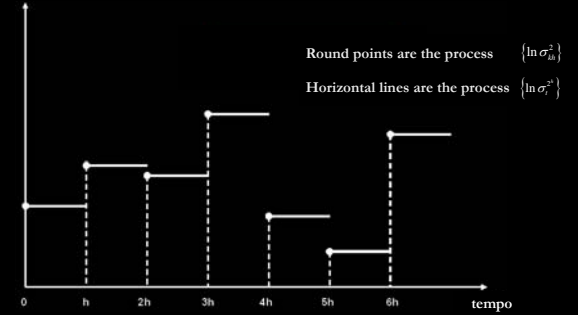
Consequently, Condition 4 is verified too

key point

From the Diffusion Limit of the M-GARCH(1,1) Process it is possible to establish a Predictive Interval for σ_t

proof

step 2: construction of the process $\{\ln \sigma_t^{2h}\}$



statement

from the M-GARCH(1,1)

$$\ln \sigma_{k+1}^2 - \ln \sigma_k^2 = \beta_0^{(k)} + (\beta_1^{(k)} - 1) \ln \sigma_k^2 + \beta_1^{(k)} \ln Z_k^2$$

Z_k are i.i.d. $N(0,1)$

Weak Convergence Theorem

$$d \ln \sigma_t^2 = (\beta_0 + 2\beta_1 E(\ln |Z_t|) + (\beta_1 - 1) \ln \sigma_t^2) dt + 2|\beta_1| \sqrt{\text{Var}(\ln |Z_t|)} dW_t^*$$

Z_t is $N(0,1)$

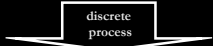
distributional properties of the S.D.E. of the M-GARCH(1,1)

$$d \ln \sigma_t^2 = [\beta_0 + 2\beta_1 E(\ln |Z_t|) + (\beta_1 - 1) \ln \sigma_t^2] dt + 2|\beta_1| \sqrt{\text{Var}(\ln |Z_t|)} dW_t^*$$

O-U process

$$\ln \sigma_t^2 \sim N \left[\left(\ln \sigma_{t-1}^2 + \frac{\beta_0 + 2\beta_1 E(\ln |Z_t|)}{(\beta_1 - 1)} \right) e^{(\beta_1 - 1)(t - t-1)} - \frac{\beta_0 + 2\beta_1 E(\ln |Z_t|)}{(\beta_1 - 1)}, \sqrt{\frac{2|\beta_1| \text{Var}(\ln |Z_t|)}{2(\beta_1 - 1)}} (e^{2(\beta_1 - 1)(t - t-1)} - 1) \right]$$

matching of the first two conditional moments



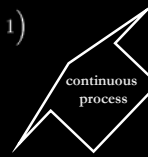
$$E(\ln \sigma_k^2) = \beta_0^{(k)} + \beta_1^{(k)} \ln \sigma_{k-1}^2 + 2\beta_1^{(k)} E(\ln |Z_{k-1}|)$$

$$Var(\ln \sigma_k^2) = 4 \left(\beta_1^{(k)} \right)^2 Var(\ln |Z_{k-1}|)$$

matching of the first two conditional moments

$$E(\ln \sigma_t^2) = \left(\ln \sigma_{t-1}^2 + \frac{\beta_0 + 2\beta_1 E(\ln |Z_t|)}{(\beta_1 - 1)} \right) e^{(\beta_1 - 1)} - \frac{\beta_0 + 2\beta_1 E(\ln |Z_t|)}{(\beta_1 - 1)}$$

$$Var(\ln \sigma_t^2) = \frac{4\beta_1^2 Var(\ln |Z_t|)}{2(\beta_1 - 1)} (e^{2(\beta_1 - 1)} - 1)$$



matching of the first two conditional moments



$$|\beta_1^{(k)}| = |\beta_1| \sqrt{\frac{e^{2(\beta_1 - 1)}}{2(\beta_1 - 1)}}$$

$$\beta_0^{(k)} = -2|\beta_1| \sqrt{\frac{e^{2(\beta_1 - 1)}}{2(\beta_1 - 1)}} E(\ln |Z_{k-1}|) - |\beta_1| \sqrt{\frac{e^{2(\beta_1 - 1)}}{2(\beta_1 - 1)}} \ln \sigma_{k-1}^2 + e^{(\beta_1 - 1)} \ln \sigma_{k-1}^2 + \frac{[\beta_0 + 2\beta_1 E(\ln |Z_{k-1}|)] (e^{(\beta_1 - 1)} - 1)}{\beta_1 - 1}$$

matching of the first two conditional moments



$$\ln \sigma_k^2 - \ln \sigma_{k-1}^2 = \frac{[\beta_0 + 2\beta_1 E(\ln |Z_{k-1}|)] (e^{(\beta_1 - 1)} - 1)}{\beta_1 - 1} - 2|\beta_1| \sqrt{\frac{e^{2(\beta_1 - 1)}}{2(\beta_1 - 1)}} E(\ln |Z_{k-1}|) + (e^{(\beta_1 - 1)} - 1) \ln \sigma_{k-1}^2 + 2|\beta_1| \sqrt{\frac{e^{2(\beta_1 - 1)}}{2(\beta_1 - 1)}} \ln |Z_{k-1}|$$

maximum likelihood estimation



$$Y_k = \ln \sigma_k^2 - \ln \sigma_{k-1}^2$$

$$a = \frac{[\beta_0 + 2\beta_1 E(\ln |Z_{k-1}|)] (e^{(\beta_1 - 1)} - 1)}{\beta_1 - 1} - E(\ln |Z_{k-1}|) |\beta_1| \sqrt{\frac{2(e^{2(\beta_1 - 1)} - 1)}{(\beta_1 - 1)}}$$

$$b = (e^{(\beta_1 - 1)} - 1)$$

$$c = |\beta_1| \sqrt{\frac{2(e^{2(\beta_1 - 1)} - 1)}{(\beta_1 - 1)}}$$

$$X_{k-1} = \ln \sigma_{k-1}^2$$

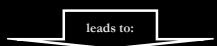
$$Z = \ln |Z_{k-1}|$$

maximum likelihood estimation

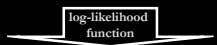


$$Y_k = a + bX_{k-1} + cZ$$

maximum likelihood estimation



$$Y_k = a + bX_{k-1} + cZ$$



$$\ln L(Y; \beta_0, \beta_1) = n \ln \left(\frac{2}{c\sqrt{2\pi}} \right) + \sum_{k=1}^n \left(\frac{Y_k - a - bX_{k-1}}{c} - \frac{1}{2} e^{2 \left(\frac{Y_k - a - bX_{k-1}}{c} \right)^2} \right)$$

maximum likelihood estimation

$$\hat{\beta}_0$$

is given, solving numerically the F.O.C.

$$\frac{\partial}{\partial \beta_0} L(Y; \beta_0, \beta_1) = 0$$

$$\hat{\beta}_1$$

is given, solving numerically the F.O.C.

$$\frac{\partial}{\partial \beta_1} L(Y; \beta_0, \beta_1) = 0$$

$$P \left(-z_{\frac{\alpha}{2}} \sqrt{\frac{(2|\beta_1| \sqrt{Var(\ln |Z_t|)})^2}{2(\beta_1 - 1)} (e^{2(\beta_1 - 1)} - 1)} + \left(\ln \sigma_{t-1}^2 + \frac{\beta_0 + 2\beta_1 E(\ln |Z_t|)}{(\beta_1 - 1)} \right) e^{(\beta_1 - 1)} - \frac{\beta_0 + 2\beta_1 E(\ln |Z_t|)}{(\beta_1 - 1)} \right) \leq \ln \sigma_t^2 \leq \left(\frac{(2|\beta_1| \sqrt{Var(\ln |Z_t|)})^2}{2(\beta_1 - 1)} (e^{2(\beta_1 - 1)} - 1) + \left(\ln \sigma_{t-1}^2 + \frac{\beta_0 + 2\beta_1 E(\ln |Z_t|)}{(\beta_1 - 1)} \right) e^{(\beta_1 - 1)} - \frac{\beta_0 + 2\beta_1 E(\ln |Z_t|)}{(\beta_1 - 1)} \right) \right) = \alpha$$

$$P \left(\begin{aligned} & -z_{\frac{\alpha}{2}} \sqrt{\frac{(2\beta_1 \sqrt{\text{Var}(\ln|\tilde{z}_t|)})^2}{2(\beta_1-1)} (e^{2(\beta_1-1)} - 1) + (\ln \sigma_{t-1}^2 + \frac{\beta_2 + 2\beta_1 \mathbb{E}(\ln|\tilde{z}_t|)}{\beta_1-1}) e^{(\beta_1-1)} - \frac{\beta_2 - 2\beta_1 \mathbb{E}(\ln|\tilde{z}_t|)}{\beta_1-1}}}{2} \leq \ln \sigma_t^2 \leq \\ & z_{\frac{\alpha}{2}} \sqrt{\frac{(2\beta_1 \sqrt{\text{Var}(\ln|\tilde{z}_t|)})^2}{2(\beta_1-1)} (e^{2(\beta_1-1)} - 1) + (\ln \sigma_{t-1}^2 + \frac{\beta_2 + 2\beta_1 \mathbb{E}(\ln|\tilde{z}_t|)}{\beta_1-1}) e^{(\beta_1-1)} - \frac{\beta_2 - 2\beta_1 \mathbb{E}(\ln|\tilde{z}_t|)}{\beta_1-1}}}{2} \end{aligned} \right) = \alpha$$

hence:

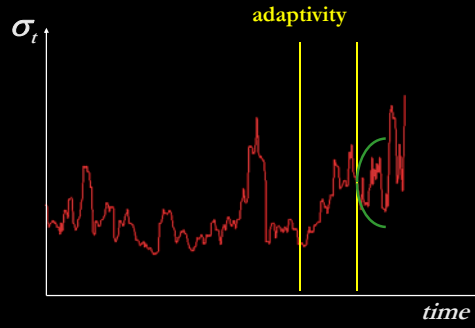
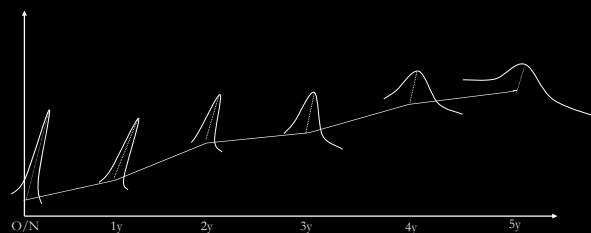
$$[\sigma_{t,\min}^G, \sigma_{t,\max}^G] = \left[e^{-\frac{z_{\frac{\alpha}{2}} \sqrt{\frac{(2.2214|\beta_1|)^2}{2(\beta_1-1)} (e^{2(\beta_1-1)} - 1) + (\ln \sigma_{t-1}^2 + \frac{\beta_2 - 1.2704\beta_1}{\beta_1-1}) e^{(\beta_1-1)} - \frac{\beta_2 - 1.2704\beta_1}{\beta_1-1}}}{2}}, e^{\frac{z_{\frac{\alpha}{2}} \sqrt{\frac{(2.2214|\beta_1|)^2}{2(\beta_1-1)} (e^{2(\beta_1-1)} - 1) + (\ln \sigma_{t-1}^2 + \frac{\beta_2 - 1.2704\beta_1}{\beta_1-1}) e^{(\beta_1-1)} - \frac{\beta_2 - 1.2704\beta_1}{\beta_1-1}}}{2}} \right]$$

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The Grid of Volatility Intervals: Loss Intervals

given the risk-free yield curve and the associated volatility surface...

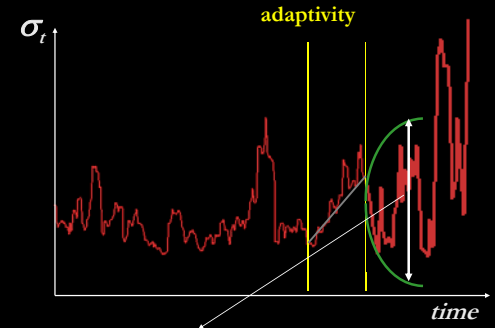
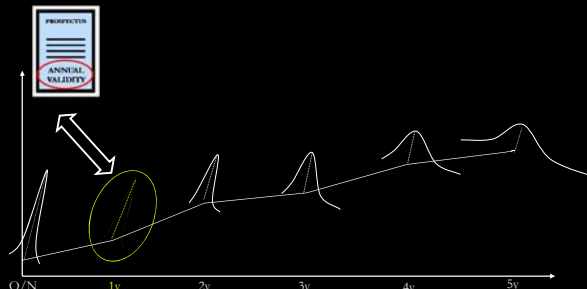


The Grid of Volatility Intervals: Loss Intervals

what is the loss in a financial investment?

The Grid of Volatility Intervals: Loss Intervals

... the probability distribution of the 1-yr risk-free rate is selected...



Width of the Predictive Interval

The Grid of Volatility Intervals: Loss Intervals

what is the loss in a financial investment?

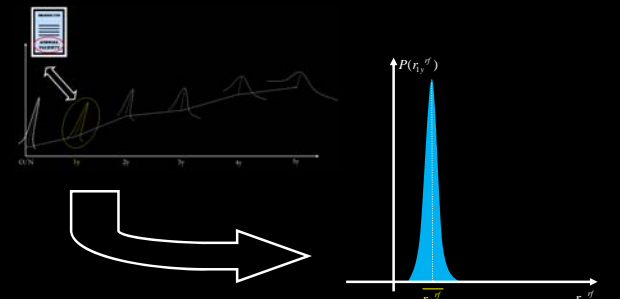


$$\text{LOSS} \in (-100\%, \overline{r^{rf}}]$$

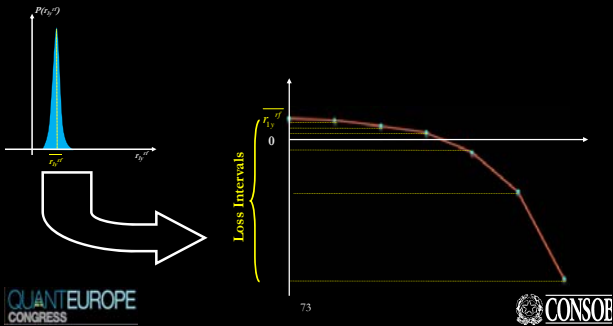
$\overline{r^{rf}}$ = average of the probability distribution of the risk-free rate

The Grid of Volatility Intervals: Loss Intervals

... the probability distribution of the 1-yr risk-free rate is selected...



...to each risk class is associated the corresponding annual loss interval (multiple of $r_{i,t}^{\theta}$ according to an exponential function)



identification of six initial loss intervals

| Risk Classes | Loss Intervals | |
|--------------|--------------------|--------------------|
| | L_{min} | L_{max} |
| low | $\theta L_{1,min}$ | $\theta L_{1,max}$ |
| medium-low | $\theta L_{2,min}$ | $\theta L_{2,max}$ |
| medium | $\theta L_{3,min}$ | $\theta L_{3,max}$ |
| medium-high | $\theta L_{4,min}$ | $\theta L_{4,max}$ |
| high | $\theta L_{5,min}$ | $\theta L_{5,max}$ |
| very high | $\theta L_{6,min}$ | $\theta L_{6,max}$ |

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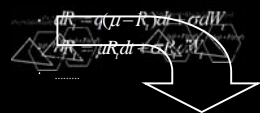
- The Grid of Volatility Intervals
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| Risk Classes | Loss Intervals | |
|--------------|--------------------|--------------------|
| | L_{min} | L_{max} |
| low | $\theta L_{1,min}$ | $\theta L_{1,max}$ |
| medium-low | $\theta L_{2,min}$ | $\theta L_{2,max}$ |
| medium | $\theta L_{3,min}$ | $\theta L_{3,max}$ |
| medium-high | $\theta L_{4,min}$ | $\theta L_{4,max}$ |
| high | $\theta L_{5,min}$ | $\theta L_{5,max}$ |
| very high | $\theta L_{6,min}$ | $\theta L_{6,max}$ |

| Risk Classes | Loss Intervals | |
|--------------|--------------------|--------------------|
| | L_{min} | L_{max} |
| low | $\theta L_{1,min}$ | $\theta L_{1,max}$ |
| medium-low | $\theta L_{2,min}$ | $\theta L_{2,max}$ |
| medium | $\theta L_{3,min}$ | $\theta L_{3,max}$ |
| medium-high | $\theta L_{4,min}$ | $\theta L_{4,max}$ |
| high | $\theta L_{5,min}$ | $\theta L_{5,max}$ |
| very high | $\theta L_{6,min}$ | $\theta L_{6,max}$ |



| Risk Classes | Volatility Intervals | |
|--------------|-------------------------|-------------------------|
| | σ_{min} | σ_{max} |
| low | $\theta \sigma_{1,min}$ | $\theta \sigma_{1,max}$ |
| medium-low | $\theta \sigma_{2,min}$ | $\theta \sigma_{2,max}$ |
| medium | $\theta \sigma_{3,min}$ | $\theta \sigma_{3,max}$ |
| medium-high | $\theta \sigma_{4,min}$ | $\theta \sigma_{4,max}$ |
| high | $\theta \sigma_{5,min}$ | $\theta \sigma_{5,max}$ |
| very high | $\theta \sigma_{6,min}$ | $\theta \sigma_{6,max}$ |

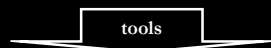
*The subscript θ preceding the volatility indicates that this is the initial interval, i.e. before the calibration

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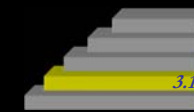
Diffusive GARCH Models

Stochastic Non-Linear Programming



Selection of an initial Volatility Interval

| Risk Classes | Volatility Intervals | |
|--------------|-------------------------|-------------------------|
| | σ_{min} | σ_{max} |
| low | $\theta \sigma_{1,min}$ | $\theta \sigma_{1,max}$ |
| medium-low | $\theta \sigma_{2,min}$ | $\theta \sigma_{2,max}$ |
| medium | $\theta \sigma_{3,min}$ | $\theta \sigma_{3,max}$ |
| medium-high | $\theta \sigma_{4,min}$ | $\theta \sigma_{4,max}$ |
| high | $\theta \sigma_{5,min}$ | $\theta \sigma_{5,max}$ |
| very high | $\theta \sigma_{6,min}$ | $\theta \sigma_{6,max}$ |



Simulation of the Fund pattern



NAV Stochastic Differential Equation



NAV S.D.E.



What Parameters?



NAV S.D.E.



What Parameters?

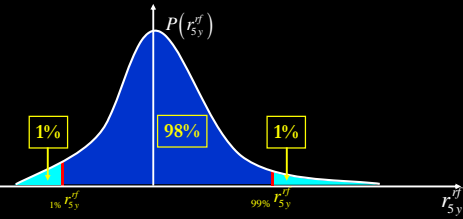
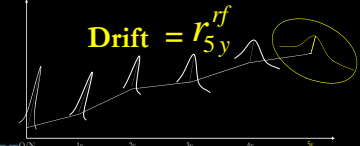
The Drift



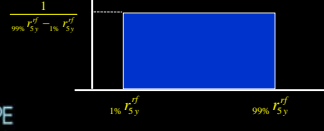
risk-neutrality principle

robustness of volatility intervals

Drift = r_{5y}^{rf}



Continuous Uniform Probability Distribution



NAV S.D.E.



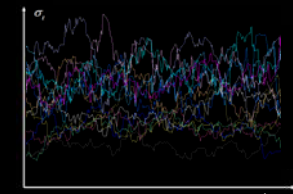
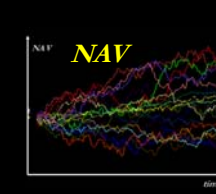
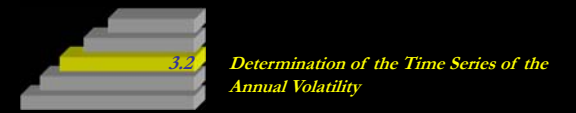
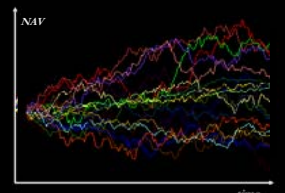
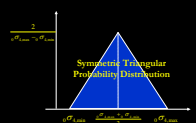
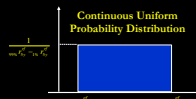
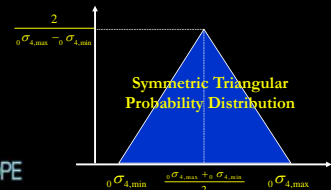
What Parameters?

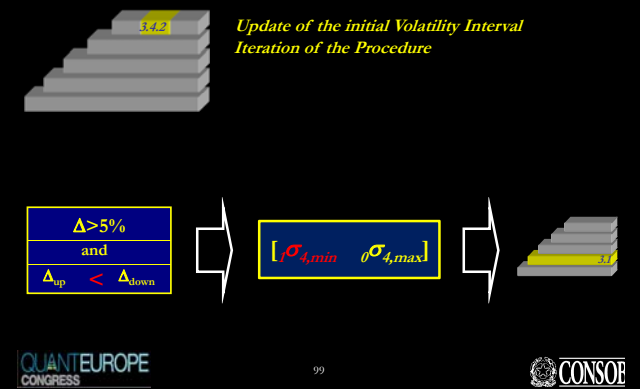
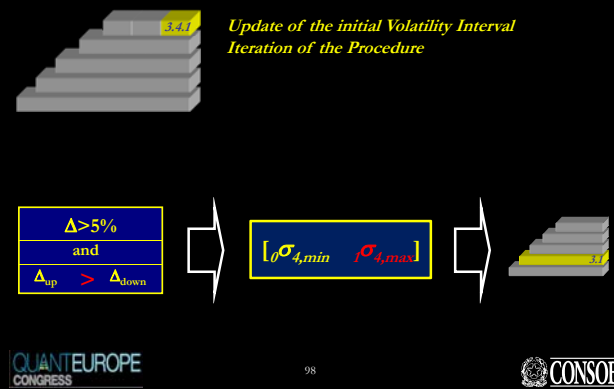
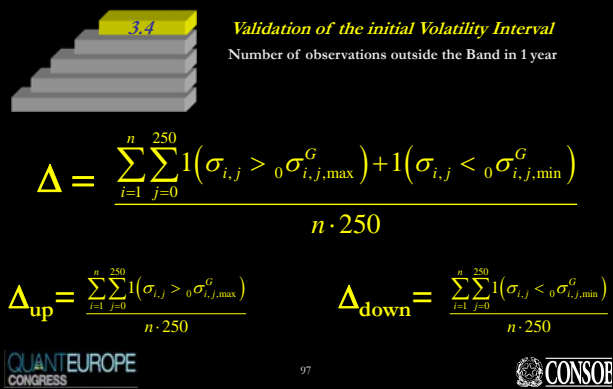
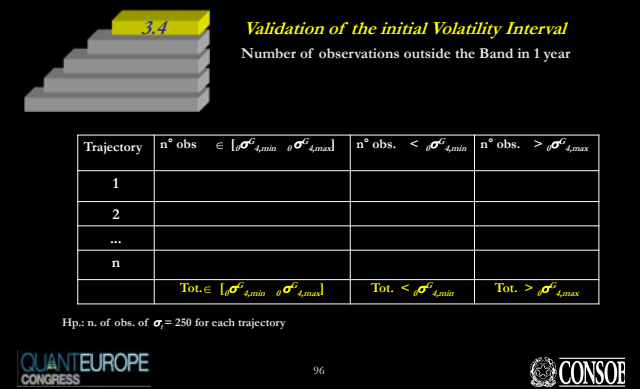
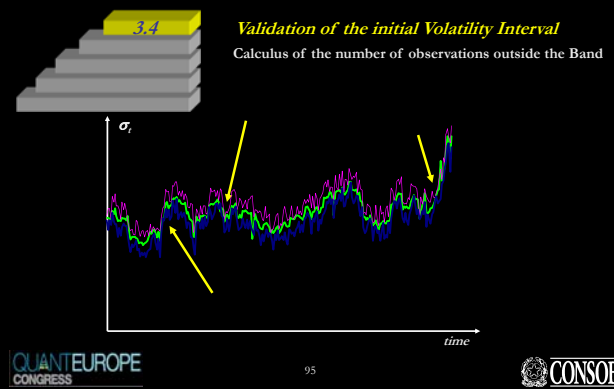
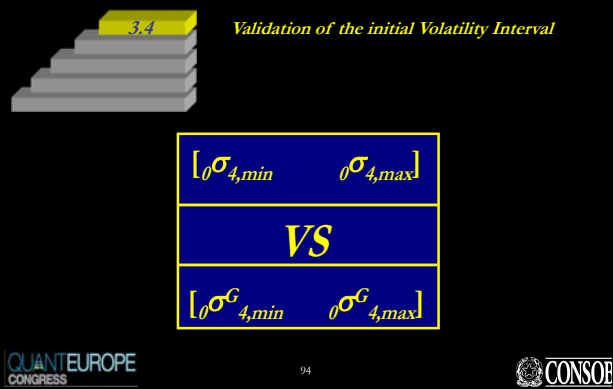
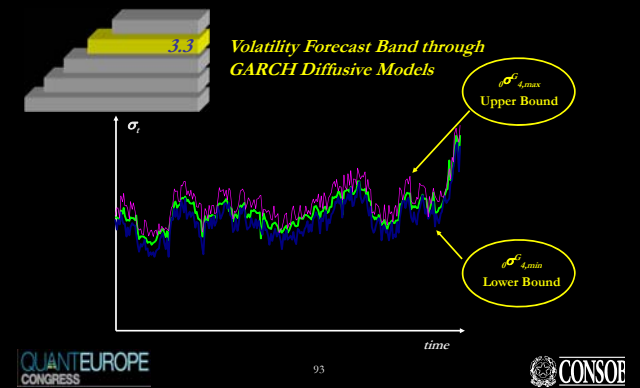
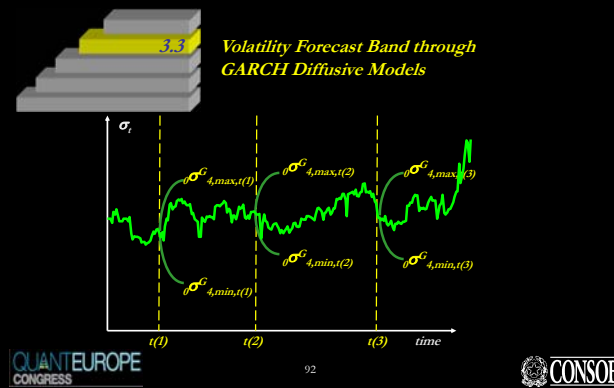
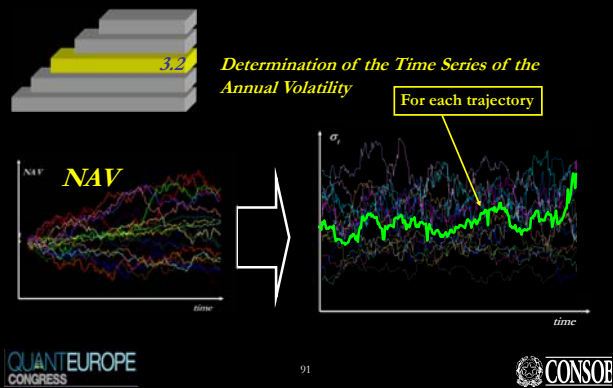
The Diffusion

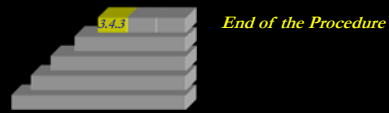


$[\sigma_{4,min}, \sigma_{4,max}]$

representativeness of volatility intervals







$$\Delta \leq 5\%$$

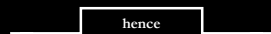
$$\begin{bmatrix} k\sigma_{4,min} & k\sigma_{4,max} \\ \sigma_{4,min} & \sigma_{4,max} \end{bmatrix} =$$

k = n. di iterations done

the iterative procedure guarantees that a fund belonging to a given risk class does not breach the GARCH adaptive band more than 5% of the days in 1 yr



migration risk is measured against fixed volatility intervals



output intervals are inherently prudential w.r.t. the 3 months migration rule

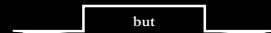
output volatility intervals

| Risk Classes | Volatility Intervals <small>(annualized values)</small> | |
|--------------|--|----------------|
| | σ_{min} | σ_{max} |
| low | 0.01% | 0.49% |
| medium-low | 0.50% | 1.59% |
| medium | 1.60% | 3.99% |
| medium-high | 4.00% | 9.99% |
| high | 10.00% | 24.99% |
| very high | 25.00% | above 25.00% |

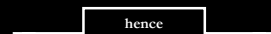
the iterative procedure guarantees that a fund belonging to a given risk class does not breach the GARCH adaptive band more than 5% of the days in 1 yr



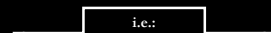
no more than 15 days over 250



migration risk is measured against fixed volatility intervals



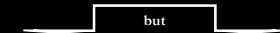
output intervals are inherently prudential w.r.t. the 3 months migration rule



output intervals are wide enough to avoid spurious migrations

Syllabus

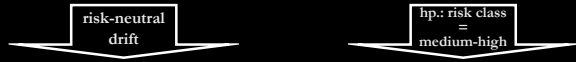
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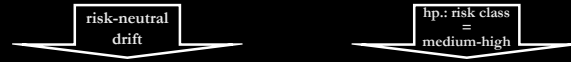
migration risk is measured against fixed volatility intervals

as confirmed by back-testing simulation

as confirmed by back-testing simulation



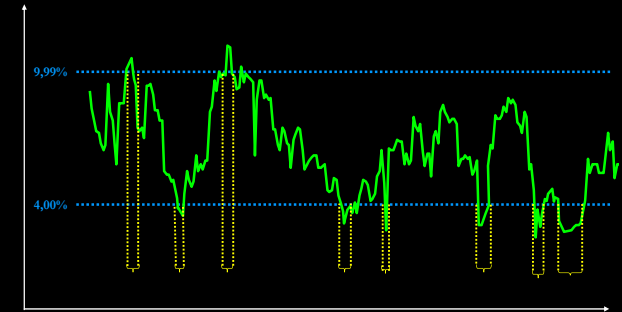
as confirmed by back-testing simulation



drawing diffusion from a symm. triang. distrib. bounded at $[\sigma_{min}, \sigma_{max}] = [4\%, 9.99\%]$ to simulate an hypothetic daily NAV and to calculate the corresponding realized volatility

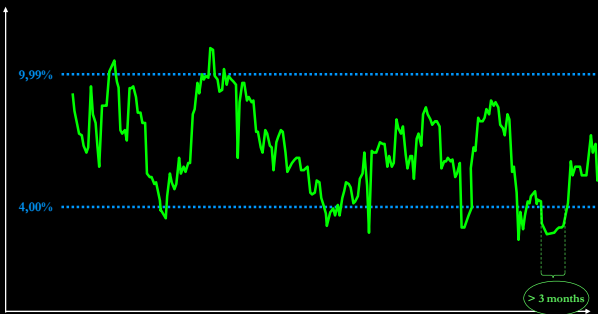
for each trajectory

as confirmed by back-testing simulation



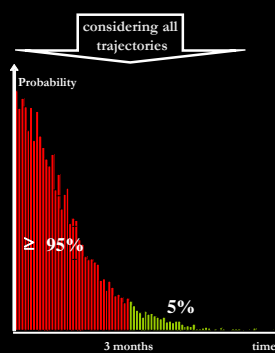
ALL THE BREACHES MARKED IN YELLOW LAST LESS THAN 3 MONTHS

as confirmed by back-testing simulation



THE OUTLIER MARKED IN GREEN LASTS MORE THAN 3 MONTHS

as confirmed by back-testing simulation



considering all trajectories

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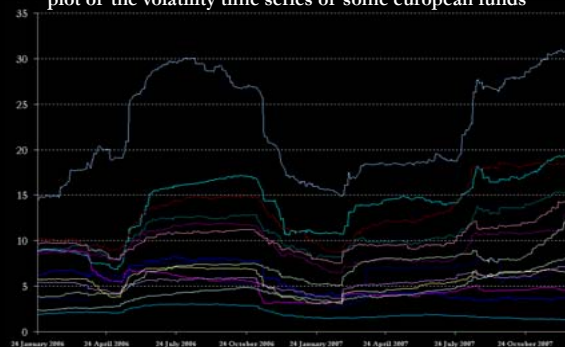
Empirical Evidence: Preliminary Informations

universe VS sample

| Country | Total (A) | Selected (B) | Representativity (B/A) |
|--------------|------------|--------------|------------------------|
| Austria | 17 | 13 | 76.5% |
| France | 92 | 53 | 57.6% |
| Germany | 63 | 45 | 71.4% |
| Ireland | 2 | 1 | 50.0% |
| Italy | 58 | 52 | 89.7% |
| Luxembourg | 252 | 153 | 60.7% |
| Spain | 224 | 130 | 58.0% |
| UK | 8 | 7 | 87.5% |
| Total | 716 | 454 | 63.4% |

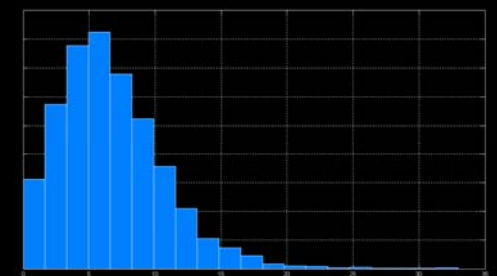
Empirical Evidence: Preliminary Informations

plot of the volatility time series of some european funds



Empirical Evidence: Preliminary Informations

volatility histogram for the sampled funds



initial distribution of the 454 funds between the 6 risk classes
(abs. values)

| Country | Initial Risk Class as from 1st January 2006 | | | | | Total |
|--------------|---|-----------|-----------|------------|-----------|------------|
| | 1 | 2 | 3 | 4 | 5 | |
| Austria | 0 | 0 | 4 | 8 | 1 | 13 |
| France | 0 | 2 | 9 | 37 | 5 | 53 |
| Germany | 0 | 2 | 10 | 26 | 7 | 45 |
| Ireland | 0 | 1 | 0 | 0 | 0 | 1 |
| Italy | 1 | 11 | 11 | 28 | 1 | 52 |
| Luxembourg | 1 | 6 | 30 | 100 | 16 | 153 |
| Spain | 0 | 23 | 33 | 62 | 12 | 130 |
| UK | 0 | 0 | 0 | 5 | 2 | 7 |
| Total | 2 | 45 | 97 | 266 | 44 | 454 |

initial distribution of the 454 funds between the 6 risk classes
(perc. values)

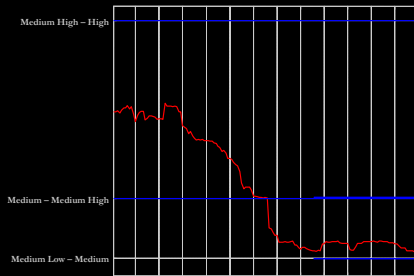
| Country | Initial Risk Class as from 1st January 2006 | | | | | Total |
|--------------|---|-------------|--------------|--------------|-------------|-------------|
| | 1 | 2 | 3 | 4 | 5 | |
| Austria | 0.0% | 0.0% | 30.8% | 61.5% | 7.7% | 100% |
| France | 0.0% | 3.8% | 17.0% | 69.8% | 9.4% | 100% |
| Germany | 0.0% | 4.4% | 22.2% | 57.8% | 15.6% | 100% |
| Ireland | 0.0% | 100.0% | 0.0% | 0.0% | 0.0% | 100% |
| Italy | 1.9% | 21.2% | 21.2% | 53.8% | 1.9% | 100% |
| Luxembourg | 0.7% | 3.9% | 19.6% | 65.4% | 10.5% | 100% |
| Spain | 0.0% | 17.7% | 25.4% | 47.7% | 9.2% | 100% |
| UK | 0.0% | 0.0% | 0.0% | 71.4% | 28.6% | 100% |
| Total | 0.4% | 9.9% | 21.4% | 58.6% | 9.7% | 100% |

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MIGRATION



— Risk Class as from the Prospectus
— Risk effectively taken

n. of migrations between different risk classes: 01.01.2006 – 12.31.2007
(abs. values)

| Country | Number of Migrations over the period January 2006 - December 2007 | | | | | | Total |
|--------------|---|-----------|------------|-----------|----------|----------|------------|
| | 0 | 1 | 2 | 3 | 4 | 5 | |
| Austria | 2 | 6 | 2 | 3 | 0 | 0 | 13 |
| France | 20 | 13 | 8 | 11 | 1 | 0 | 53 |
| Germany | 18 | 6 | 13 | 8 | 0 | 0 | 45 |
| Ireland | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| Italy | 15 | 12 | 17 | 8 | 0 | 0 | 52 |
| Luxembourg | 63 | 28 | 34 | 23 | 4 | 1 | 153 |
| Spain | 44 | 30 | 31 | 21 | 4 | 0 | 130 |
| UK | 1 | 3 | 1 | 2 | 0 | 0 | 7 |
| Total | 163 | 99 | 106 | 76 | 9 | 1 | 454 |

migrations per Country: 01.01.2006 – 12.31.2007
(perc. values)

| Country | 0 | 1 | 2 | 3 | 4 | 5 | Total |
|------------|-------|--------|-------|-------|------|------|-------|
| Austria | 15.4% | 46.2% | 15.4% | 23.1% | 0.0% | 0.0% | 100% |
| France | 37.7% | 24.5% | 15.1% | 20.8% | 1.9% | 0.0% | 100% |
| Germany | 40.0% | 13.3% | 28.9% | 17.8% | 0.0% | 0.0% | 100% |
| Ireland | 0.0% | 100.0% | 0.0% | 0.0% | 0.0% | 0.0% | 100% |
| Italy | 28.8% | 23.1% | 32.7% | 15.4% | 0.0% | 0.0% | 100% |
| Luxembourg | 41.2% | 18.3% | 22.2% | 15.0% | 2.6% | 0.7% | 100% |
| Spain | 33.8% | 23.1% | 23.8% | 16.2% | 3.1% | 0.0% | 100% |
| UK | 14.3% | 42.9% | 14.3% | 28.6% | 0.0% | 0.0% | 100% |

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Conclusions

- ✓ REGULATORY ISSUE:
measurement and representation of mutual funds risk profile
- ✓ ADEQUACY REQUIREMENT OF THE APPROACH:
ability to pursue full risk disclosure consistently with the “standard” activity of fund managers
- ✓ DIFFUSIVE GARCH:
definition of adaptive volatility prediction bands
- ✓ EMPIRICAL EVIDENCE:
the phenomenon of the migration interests more the funds belonging to the riskiest classes
- ✓ CLOSING RECOMMENDATION:
exploring other fields of application, especially to move faster towards a really levelled playing field

QUANTEUROPE CONGRESS

CONSOB

VOLATILITY METRICS TO ASSESS RELATIVE RISK IN THE
QUANTITATIVE PORTFOLIO MANAGEMENT OF MUTUAL FUNDS:
A REGULATORY APPROACH BASED ON DIFFUSIVE GARCH

LONDON, NOVEMBER 6TH 2008

MARCELLO MINENNA
GIOVANNA MARIA BOI