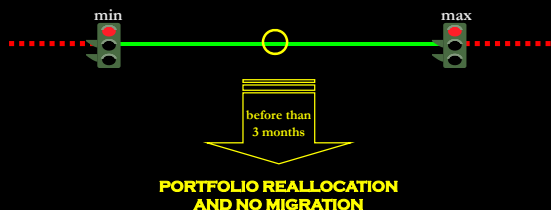
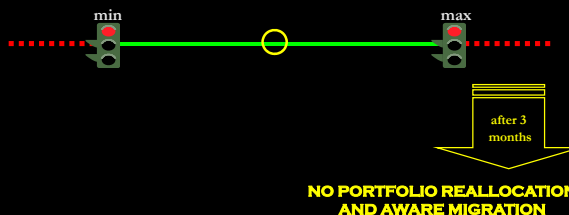


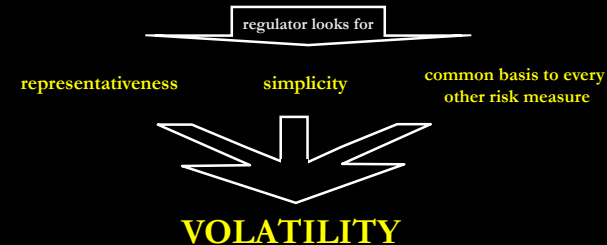
wide enough intervals do imply aware migration



wide enough intervals do imply aware migration



a bunch of risk metrics in finance: which one to choose?

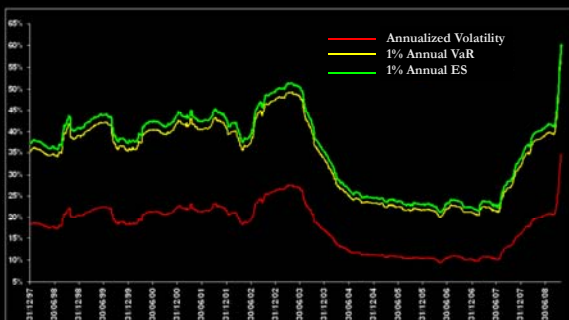


a bunch of volatility measures: which one to choose?



Annualized Volatility of Daily NAV Returns

1:1 correspondance with other risk measures



Syllabus

- The Risk Profile of Mutual Funds
 - Representation via Qualitative Risk Classes
 - Adequate Underlying Risk Metric
- Diffusive GARCH
 - Intuition
 - The Weak Convergence Theorem on R^2
 - The Continuous Limit of the M-GARCH(1,1)
 - The Prediction Interval for the Volatility
- The Grid of Volatility Intervals
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 - Intervals Fine-Tuning: the Iterative Procedure
 - Intervals Adequacy
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 - The Evolution of the Risk Profile over Time

Conclusions

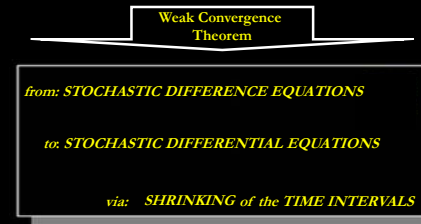
need for volatility forecasts

need for volatility forecasts

through the diffusion limit of GARCH

need for volatility forecasts

through the diffusion limit of GARCH



statement

The sequence $\{X_t^h\}$, whose measurable space is $(\mathbb{R}^2, \mathcal{B}(\mathbb{R}^2))$, converges weakly for $h \downarrow 0$ to the process $\{X_t\}$ which has a unique distribution and is characterized by the following stochastic differential equation:

$$dX_t = b(x, t)dt + \sigma(x, t)dW_{2,t}$$

where $W_{2,t}$ is a two-dimensional standard Brownian motion, if the conditions 1-4, presented below, are satisfied.

statement

The process $\{X_t\}$ has a distribution independent on the choice of $\sigma(x, t)$ and it takes finite values over finite time intervals, i.e. $\forall T > 0$:

$$P\left(\sup_{0 \leq t \leq T} \|X_t\| < \infty\right) = 1$$

conditions:

n. 1

If \exists a $\delta > 0$ s.t.:

$$\lim_{h \downarrow 0} \begin{pmatrix} c_{h,\delta}(x_1, t) \\ c_{h,\delta}(x_2, t) \end{pmatrix} = 0$$

conditions:

n. 1

If \exists a $\delta > 0$ s.t.:

$$\lim_{h \downarrow 0} \begin{pmatrix} c_{h,\delta}(x_1, t) \\ c_{h,\delta}(x_2, t) \end{pmatrix} = 0$$

then \exists

$$a(x, t) \lim_{h \downarrow 0} \begin{pmatrix} a_h(x_1, t) & a_h((x_1, x_2), t) \\ a_h((x_2, x_1), t) & a_h(x_2, t) \end{pmatrix} = \begin{pmatrix} a(x_1, t) & 0 \\ 0 & a(x_2, t) \end{pmatrix}$$

s.t.

$$b(x, t) \lim_{h \downarrow 0} \begin{pmatrix} b_h(x_1, t) \\ b_h(x_2, t) \end{pmatrix} = \begin{pmatrix} b(x_1, t) \\ b(x_2, t) \end{pmatrix}$$

conditions:

n. 2

$\exists \sigma(x, t)$ s.t.: $\forall x_1 \in \mathbb{R}^1, \forall x_2 \in \mathbb{R}^1$,

it holds

$$\begin{pmatrix} \sigma(x_1, t) & 0 \\ 0 & \sigma(x_2, t) \end{pmatrix} = \begin{pmatrix} \sqrt{a(x_1, t)} & 0 \\ 0 & \sqrt{a(x_2, t)} \end{pmatrix}$$

conditions:

n. 3

For $h \downarrow 0$, X_0^h converges in distribution to a random variable X_0 with probability measure ν_0 on $(\mathbb{R}^2, \mathcal{B}(\mathbb{R}^2))$

n. 4

ν_0 , $a(x, t)$ and $b(x, t)$ uniquely specify the distribution of the process $\{X_t\}$ characterized by an initial distribution ν_0 , a conditional second moment $a(x, t)$ and a conditional first moment $b(x, t)$

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statement

from the M-GARCH(1,1)

$$\begin{cases} X_k - X_{k-1} = \gamma \cdot (\eta - X_{k-1}) + \sigma_k \bar{Z}_k \\ \text{and} \\ \ln \sigma_{k+1}^2 - \ln \sigma_k^2 = \beta_0^{(k)} + (\beta_1^{(k)} - 1) \ln \sigma_k^2 + \beta_1^{(k)} \ln Z_k^2 \\ \text{or, equivalently} \\ \ln \sigma_{k+1}^2 - \ln \sigma_k^2 = \beta_0^{(k)} + (\beta_1^{(k)} - 1) \ln \sigma_k^2 + 2\beta_1^{(k)} \ln |Z_k| \end{cases}$$

\bar{Z}_k and Z_k are i.i.d. $N(0,1)$

statement

from the M-GARCH(1,1)

$$\begin{cases} X_k - X_{k-1} = \gamma \cdot (\eta - X_{k-1}) + \sigma_k \bar{Z}_k \\ \text{and} \\ \ln \sigma_{k+1}^2 - \ln \sigma_k^2 = \beta_0^{(k)} + (\beta_1^{(k)} - 1) \ln \sigma_k^2 + \beta_1^{(k)} \ln Z_k^2 \\ \text{or, equivalently} \\ \ln \sigma_{k+1}^2 - \ln \sigma_k^2 = \beta_0^{(k)} + (\beta_1^{(k)} - 1) \ln \sigma_k^2 + 2\beta_1^{(k)} \ln |Z_k| \end{cases}$$

\bar{Z}_k and Z_k are i.i.d. $N(0,1)$

Weak Convergence Theorem

$$dX_t = q(\mu - X_t)dt + \sigma_t dW_t$$

$$d \ln \sigma_t^2 = (\beta_0 + 2\beta_1 E(\ln |Z_t|) + (\beta_1 - 1) \ln \sigma_t^2) dt + 2|\beta_1| \sqrt{\text{Var}(\ln |Z_t|)} dW_t^*$$

Z_t is $N(0,1)$

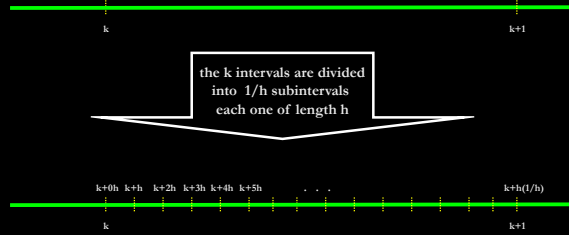
proof

step 1: rescaling of the discrete process



proof

step 1: rescaling of the discrete process



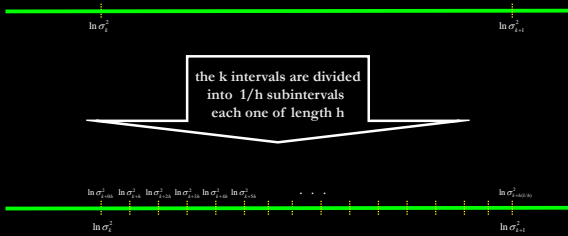
proof

step 1: rescaling of the discrete process



proof

step 1: rescaling of the discrete process



proof

step 1: rescaling of the discrete process

relationship from rescaling

$$\ln \sigma_k^2 - \ln \sigma_{k-1}^2 = \sum_{j=1}^{\frac{1}{h}} \left(\ln \sigma_{(k-1)+jh}^2 - \ln \sigma_{(k-1)+j(h-h)}^2 \right)$$

proof

step 1: rescaling of the discrete process

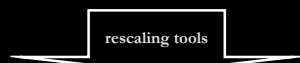
rescaling tools

$$(\ln |Z|)_{kh} = \sqrt{h} \ln |Z_{k-1}| + (h - \sqrt{h}) E(\ln |Z_{k-1}|)$$

where: $f_{Z_{kh}}(z_{kh}) = (1/\sqrt{h}) f_{Z_k}(z_{kh})$

proof

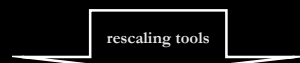
step 1: rescaling of the discrete process



$$\beta_{0h} = \beta_0^{(k)} \cdot h$$

proof

step 1: rescaling of the discrete process

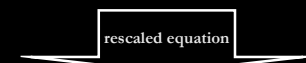


$$\beta_{1h} \text{ s.t. :}$$

$$0 = (\beta_0 + \beta_1 \ln \sigma_{k-1}^2 + 2\beta_1 \ln |Z_{k-1}|) - \beta_{1h}^{\frac{1}{h}} \ln \sigma_{k-1}^2 - (\beta_0 h + 2\beta_1 h (\sqrt{h} \ln |Z_{k-1}| + (h - \sqrt{h}) E(\ln |Z_{k-1}|))) \sum_{j=1}^{\frac{1}{h}} \beta_{1h}^j$$

proof

step 1: rescaling of the discrete process



$$\ln \sigma_{kh}^2 = \beta_{0h} + \beta_{1h} \ln \sigma_{(k-1)h}^2 + 2\beta_{1h} (\sqrt{h} \ln |Z_{k-1}| + (h - \sqrt{h}) E(\ln |Z_{k-1}|))$$

proof

step 2: construction of the process $\{\ln \sigma_t^{2^h}\}$

definition of the prob. measure P_h on the Skorokhod space D s.t.:

$$P_h(\ln \sigma_0^{2^h} \in \Gamma) = \nu_0(\Gamma) \quad \forall \Gamma \in \mathcal{B}(\mathbb{R}^1)$$

$$P_h(\ln \sigma_t^{2^h} = \ln \sigma_{kh}^2, \quad \forall kh \leq t < (k+1)h) = 1$$

$$P_h(\ln \sigma_{(k+1)h}^2 \in \Gamma | \widehat{\mathcal{S}}_{kh}) = \Pi_{h,kh}(\ln \sigma_{kh}^2, \Gamma) \text{ a.s. under } P_h, \forall k \geq 0, \forall \Gamma \in \mathcal{B}(\mathbb{R}^1)$$

proof

step 3: check of condition n. 1 of the weak convergence th.

$$\beta_{0h} := \beta_0 \cdot h$$

$$\beta_{1h} := \beta_1 \cdot h$$

$$\lim_{h \downarrow 0} c_{h, \beta-1}(\widehat{\ln \sigma^2}, t) = 0$$

$$\lim_{h \downarrow 0} b_h(\widehat{\ln \sigma^2}, t) = (\beta_0 + 2\beta_1 \mathbf{E}(\ln |Z_t|) + (\beta_1 - 1) \ln \sigma_t^2)$$

$$\lim_{h \downarrow 0} a_h(\widehat{\ln \sigma^2}, t) = 4\beta_1^2 \text{Var}(\ln |Z_t|)$$

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proof

step 2: construction of the process $\{\ln \sigma_t^{2^h}\}$

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$$P_h(\ln \sigma_{(k+1)h}^2 \in \Gamma | \widehat{\mathcal{S}}_{kh}) = \Pi_{h,kh}(\ln \sigma_{kh}^2, \Gamma) \text{ a.s. under } P_h, \forall k \geq 0, \forall \Gamma \in \mathcal{B}(\mathbb{R}^1)$$

$$\ln \sigma_{t+h}^{2^h} - \ln \sigma_t^{2^h} = \beta_{0h} + (\beta_{1h} - h) \ln \sigma_t^{2^h} + 2\beta_{1h} \left\{ \sqrt{h} \ln |Z_t^h| + (h - \sqrt{h}) E(\ln |Z_t^h|) \right\}$$

proof

step 3: check of condition n. 1 of the weak convergence th.

Condition n. 2 is verified for every $\sigma > 0$, i.e.:

$$\sigma(\ln \widehat{\sigma^2}, t) = 2|\beta_1| \sqrt{\text{Var}(\ln |Z_t|)}$$

Condition n. 3 is satisfied by construction of the process $\{\ln \sigma_t^{2^h}\}$

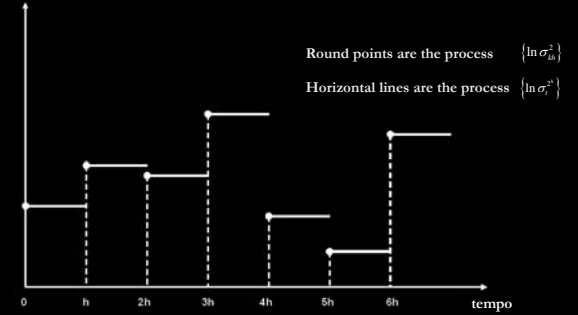
Consequently, Condition 4 is verified too

key point

From the Diffusion Limit of the M-GARCH(1,1) Process it is possible to establish a Predictive Interval for σ_t

proof

step 2: construction of the process $\{\ln \sigma_t^{2^h}\}$



statement

from the M-GARCH(1,1)

$$\ln \sigma_{k+1}^2 - \ln \sigma_k^2 = \beta_0^{(k)} + (\beta_1^{(k)} - 1) \ln \sigma_k^2 + \beta_1^{(k)} \ln Z_k^2$$

Z_k are i.i.d. $N(0,1)$

Weak Convergence Theorem

$$d \ln \sigma_t^2 = (\beta_0 + 2\beta_1 E(\ln |Z_t|) + (\beta_1 - 1) \ln \sigma_t^2) dt + 2|\beta_1| \sqrt{\text{Var}(\ln |Z_t|)} dW_t^*$$

Z_t is $N(0,1)$

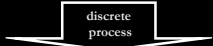
distributional properties of the S.D.E. of the M-GARCH(1,1)

$$d \ln \sigma_t^2 = [\beta_0 + 2\beta_1 E(\ln |Z_t|) + (\beta_1 - 1) \ln \sigma_t^2] dt + 2|\beta_1| \sqrt{\text{Var}(\ln |Z_t|)} dW_t^*$$

O-U process

$$\ln \sigma_t^2 \sim N \left[\left(\ln \sigma_{t-1}^2 + \frac{\beta_0 + 2\beta_1 E(\ln |Z_t|)}{(\beta_1 - 1)} \right) e^{(\beta_1 - 1)(t - t-1)} - \frac{\beta_0 + 2\beta_1 E(\ln |Z_t|)}{(\beta_1 - 1)}, \sqrt{\frac{2|\beta_1| \text{Var}(\ln |Z_t|)}{2(\beta_1 - 1)}} (e^{2(\beta_1 - 1)(t - t-1)} - 1) \right]$$

matching of the first two conditional moments



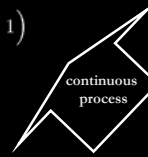
$$E(\ln \sigma_k^2) = \beta_0^{(k)} + \beta_1^{(k)} \ln \sigma_{k-1}^2 + 2\beta_1^{(k)} E(\ln |Z_{k-1}|)$$

$$Var(\ln \sigma_k^2) = 4 \left(\beta_1^{(k)} \right)^2 Var(\ln |Z_{k-1}|)$$

matching of the first two conditional moments

$$E(\ln \sigma_t^2) = \left(\ln \sigma_{t-1}^2 + \frac{\beta_0 + 2\beta_1 E(\ln |Z_t|)}{(\beta_1 - 1)} \right) e^{(\beta_1 - 1)} - \frac{\beta_0 + 2\beta_1 E(\ln |Z_t|)}{(\beta_1 - 1)}$$

$$Var(\ln \sigma_t^2) = \frac{4\beta_1^2 Var(\ln |Z_t|)}{2(\beta_1 - 1)} \left(e^{2(\beta_1 - 1)} - 1 \right)$$



matching of the first two conditional moments



$$|\beta_1^{(k)}| = |\beta_1| \sqrt{\frac{e^{2(\beta_1 - 1)}}{2(\beta_1 - 1)}}$$

$$\beta_0^{(k)} = -2|\beta_1| \sqrt{\frac{e^{2(\beta_1 - 1)}}{2(\beta_1 - 1)}} E(\ln |Z_{k-1}|) - |\beta_1| \sqrt{\frac{e^{2(\beta_1 - 1)}}{2(\beta_1 - 1)}} \ln \sigma_{k-1}^2 + e^{(\beta_1 - 1)} \ln \sigma_{k-1}^2 + \frac{[\beta_0 + 2\beta_1 E(\ln |Z_{k-1}|)] (e^{(\beta_1 - 1)} - 1)}{\beta_1 - 1}$$

matching of the first two conditional moments



$$\ln \sigma_k^2 - \ln \sigma_{k-1}^2 = \frac{[\beta_0 + 2\beta_1 E(\ln |Z_{k-1}|)] (e^{(\beta_1 - 1)} - 1)}{\beta_1 - 1} - 2|\beta_1| \sqrt{\frac{e^{2(\beta_1 - 1)}}{2(\beta_1 - 1)}} E(\ln |Z_{k-1}|) + (e^{(\beta_1 - 1)} - 1) \ln \sigma_{k-1}^2 + 2|\beta_1| \sqrt{\frac{e^{2(\beta_1 - 1)}}{2(\beta_1 - 1)}} \ln |Z_{k-1}|$$

maximum likelihood estimation



$$Y_k = \ln \sigma_k^2 - \ln \sigma_{k-1}^2$$

$$a = \frac{[\beta_0 + 2\beta_1 E(\ln |Z_{k-1}|)] (e^{(\beta_1 - 1)} - 1)}{\beta_1 - 1} - E(\ln |Z_{k-1}|) |\beta_1| \sqrt{\frac{2(e^{2(\beta_1 - 1)} - 1)}{(\beta_1 - 1)}}$$

$$b = (e^{(\beta_1 - 1)} - 1)$$

$$c = |\beta_1| \sqrt{\frac{2(e^{2(\beta_1 - 1)} - 1)}{(\beta_1 - 1)}}$$

$$X_{k-1} = \ln \sigma_{k-1}^2$$

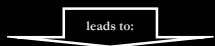
$$Z = \ln |Z_{k-1}|$$

maximum likelihood estimation

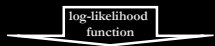


$$Y_k = a + bX_{k-1} + cZ$$

maximum likelihood estimation



$$Y_k = a + bX_{k-1} + cZ$$



$$\ln L(Y; \beta_0, \beta_1) = n \ln \left(\frac{2}{c\sqrt{2\pi}} \right) + \sum_{k=1}^n \left(\frac{Y_k - a - bX_{k-1}}{c} - \frac{1}{2} e^{2 \left(\frac{Y_k - a - bX_{k-1}}{c} \right)^2} \right)$$

maximum likelihood estimation

$$\hat{\beta}_0$$

is given, solving numerically the F.O.C.

$$\hat{\beta}_1$$

is given, solving numerically the F.O.C.

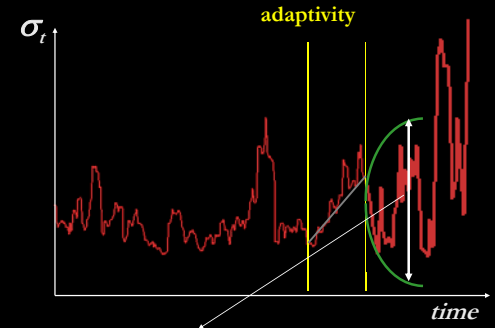
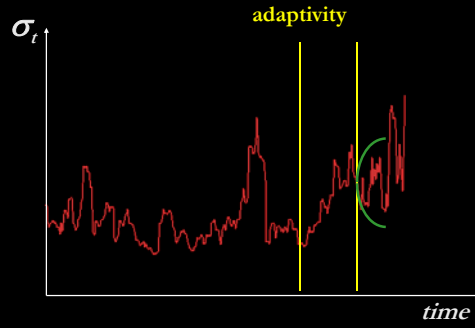
$$\frac{\partial}{\partial \beta_0} L(Y; \beta_0, \beta_1) = 0 \quad \frac{\partial}{\partial \beta_1} L(Y; \beta_0, \beta_1) = 0$$

$$P \left(-z_{\frac{\alpha}{2}} \sqrt{\frac{(2|\beta_1| \sqrt{Var(\ln |Z_t|)})^2}{2(\beta_1 - 1)} (e^{2(\beta_1 - 1)} - 1)} + \left(\ln \sigma_{t-1}^2 + \frac{\beta_0 + 2\beta_1 E(\ln |Z_t|)}{(\beta_1 - 1)} \right) e^{(\beta_1 - 1)} - \frac{\beta_0 + 2\beta_1 E(\ln |Z_t|)}{(\beta_1 - 1)} \right) \leq \ln \sigma_t^2 \leq \left(\frac{(2|\beta_1| \sqrt{Var(\ln |Z_t|)})^2}{2(\beta_1 - 1)} (e^{2(\beta_1 - 1)} - 1) + \left(\ln \sigma_{t-1}^2 + \frac{\beta_0 + 2\beta_1 E(\ln |Z_t|)}{(\beta_1 - 1)} \right) e^{(\beta_1 - 1)} - \frac{\beta_0 + 2\beta_1 E(\ln |Z_t|)}{(\beta_1 - 1)} \right) \right) = \alpha$$

$$P \left(\begin{aligned} & -z_{\frac{\alpha}{2}} \sqrt{\frac{(2\beta_1 \sqrt{\text{Var}(\ln|\tilde{z}_t|)})^2}{2(\beta_1-1)} (e^{2(\beta_1-1)} - 1) + (\ln \sigma_{t-1}^2 + \frac{\beta_2 + 2\beta_1 \mathbb{E}(\ln|\tilde{z}_t|)}{\beta_1-1}) e^{(\beta_1-1)} - \frac{\beta_2 - 2\beta_1 \mathbb{E}(\ln|\tilde{z}_t|)}{\beta_1-1}}}{2} \leq \ln \sigma_t^2 \leq \\ & z_{\frac{\alpha}{2}} \sqrt{\frac{(2\beta_1 \sqrt{\text{Var}(\ln|\tilde{z}_t|)})^2}{2(\beta_1-1)} (e^{2(\beta_1-1)} - 1) + (\ln \sigma_{t-1}^2 + \frac{\beta_2 + 2\beta_1 \mathbb{E}(\ln|\tilde{z}_t|)}{\beta_1-1}) e^{(\beta_1-1)} - \frac{\beta_2 - 2\beta_1 \mathbb{E}(\ln|\tilde{z}_t|)}{\beta_1-1}}}{2} \end{aligned} \right) = \alpha$$

hence:

$$[\sigma_{t,\min}^G, \sigma_{t,\max}^G] = \left[e^{\frac{-z_{\frac{\alpha}{2}} \sqrt{\frac{(2.2214|\beta_1|)^2}{2(\beta_1-1)} (e^{2(\beta_1-1)} - 1) + (\ln \sigma_{t-1}^2 + \frac{\beta_2 - 1.2704\beta_1}{\beta_1-1}) e^{(\beta_1-1)} - \frac{\beta_2 - 1.2704\beta_1}{\beta_1-1}}}{2}}, e^{\frac{z_{\frac{\alpha}{2}} \sqrt{\frac{(2.2214|\beta_1|)^2}{2(\beta_1-1)} (e^{2(\beta_1-1)} - 1) + (\ln \sigma_{t-1}^2 + \frac{\beta_2 - 1.2704\beta_1}{\beta_1-1}) e^{(\beta_1-1)} - \frac{\beta_2 - 1.2704\beta_1}{\beta_1-1}}}{2}} \right]$$



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The Grid of Volatility Intervals: Loss Intervals

what is the loss in a financial investment?

what is the loss in a financial investment?

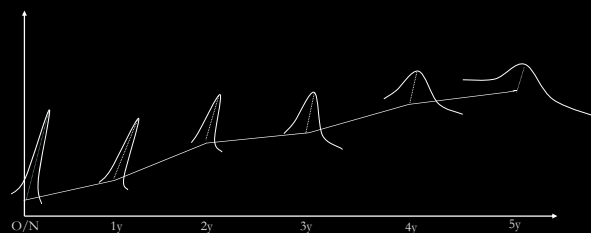


$$\text{LOSS} \in (-100\%, \overline{r^{rf}}]$$

$\overline{r^{rf}}$ = average of the probability distribution of the risk-free rate

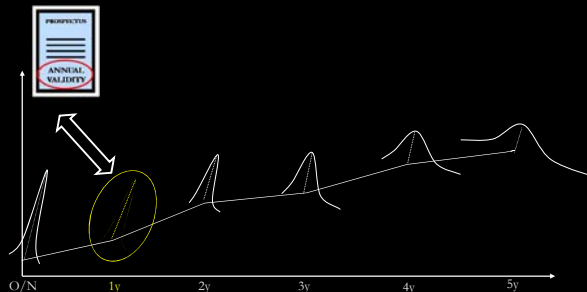
The Grid of Volatility Intervals: Loss Intervals

given the risk-free yield curve and the associated volatility surface...



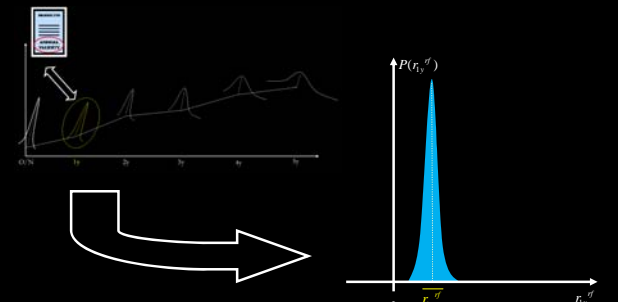
The Grid of Volatility Intervals: Loss Intervals

... the probability distribution of the 1-yr risk-free rate is selected...

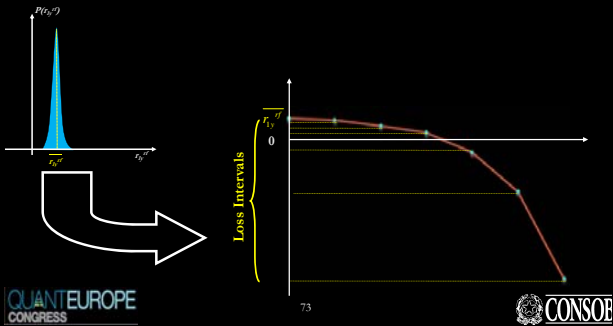


The Grid of Volatility Intervals: Loss Intervals

... the probability distribution of the 1-yr risk-free rate is selected...



...to each risk class is associated the corresponding annual loss interval (multiple of $r_{i,t}^{\theta}$ according to an exponential function)



identification of six initial loss intervals

Risk Classes	Loss Intervals	
	L_{min}	L_{max}
low	$\theta L_{1,min}$	$\theta L_{1,max}$
medium-low	$\theta L_{2,min}$	$\theta L_{2,max}$
medium	$\theta L_{3,min}$	$\theta L_{3,max}$
medium-high	$\theta L_{4,min}$	$\theta L_{4,max}$
high	$\theta L_{5,min}$	$\theta L_{5,max}$
very high	$\theta L_{6,min}$	$\theta L_{6,max}$

- The Risk Profile of Mutual Funds
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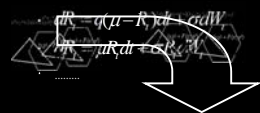
- The Grid of Volatility Intervals
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Risk Classes	Loss Intervals	
	L_{min}	L_{max}
low	$\theta L_{1,min}$	$\theta L_{1,max}$
medium-low	$\theta L_{2,min}$	$\theta L_{2,max}$
medium	$\theta L_{3,min}$	$\theta L_{3,max}$
medium-high	$\theta L_{4,min}$	$\theta L_{4,max}$
high	$\theta L_{5,min}$	$\theta L_{5,max}$
very high	$\theta L_{6,min}$	$\theta L_{6,max}$

Risk Classes	Loss Intervals	
	L_{min}	L_{max}
low	$\theta L_{1,min}$	$\theta L_{1,max}$
medium-low	$\theta L_{2,min}$	$\theta L_{2,max}$
medium	$\theta L_{3,min}$	$\theta L_{3,max}$
medium-high	$\theta L_{4,min}$	$\theta L_{4,max}$
high	$\theta L_{5,min}$	$\theta L_{5,max}$
very high	$\theta L_{6,min}$	$\theta L_{6,max}$



Risk Classes	Volatility Intervals	
	σ_{min}	σ_{max}
low	$\theta \sigma_{1,min}$	$\theta \sigma_{1,max}$
medium-low	$\theta \sigma_{2,min}$	$\theta \sigma_{2,max}$
medium	$\theta \sigma_{3,min}$	$\theta \sigma_{3,max}$
medium-high	$\theta \sigma_{4,min}$	$\theta \sigma_{4,max}$
high	$\theta \sigma_{5,min}$	$\theta \sigma_{5,max}$
very high	$\theta \sigma_{6,min}$	$\theta \sigma_{6,max}$

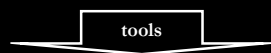
*The subscript θ preceding the volatility indicates that this is the initial interval, i.e. before the calibration

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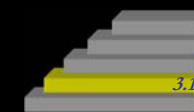
Diffusive GARCH Models

Stochastic Non-Linear Programming



Selection of an initial Volatility Interval

Risk Classes	Volatility Intervals	
	σ_{min}	σ_{max}
low	$\theta \sigma_{1,min}$	$\theta \sigma_{1,max}$
medium-low	$\theta \sigma_{2,min}$	$\theta \sigma_{2,max}$
medium	$\theta \sigma_{3,min}$	$\theta \sigma_{3,max}$
medium-high	$\theta \sigma_{4,min}$	$\theta \sigma_{4,max}$
high	$\theta \sigma_{5,min}$	$\theta \sigma_{5,max}$
very high	$\theta \sigma_{6,min}$	$\theta \sigma_{6,max}$



Simulation of the Fund pattern



NAV Stochastic Differential Equation



NAV S.D.E.



What Parameters?



NAV S.D.E.



What Parameters?

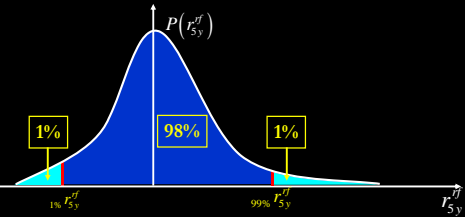
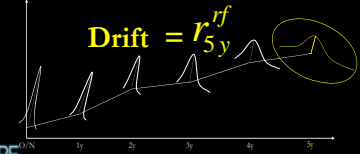
The Drift



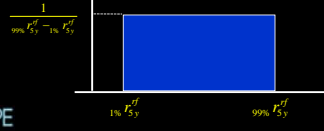
risk-neutrality principle

robustness of volatility intervals

$$\text{Drift} = r_{5y}^{rf}$$



Continuous Uniform Probability Distribution



NAV S.D.E.



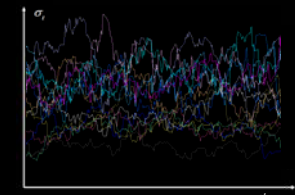
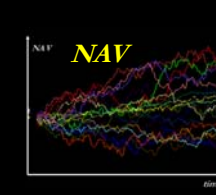
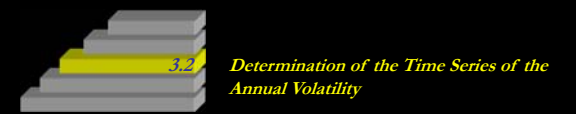
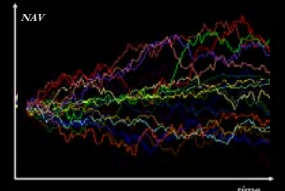
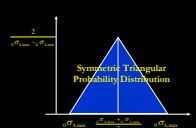
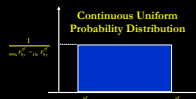
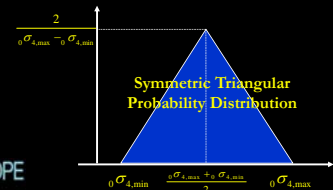
What Parameters?

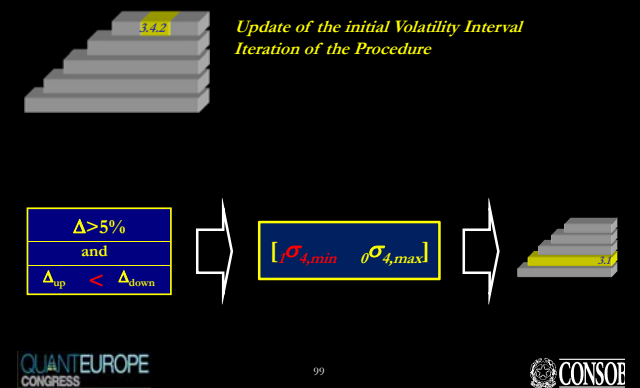
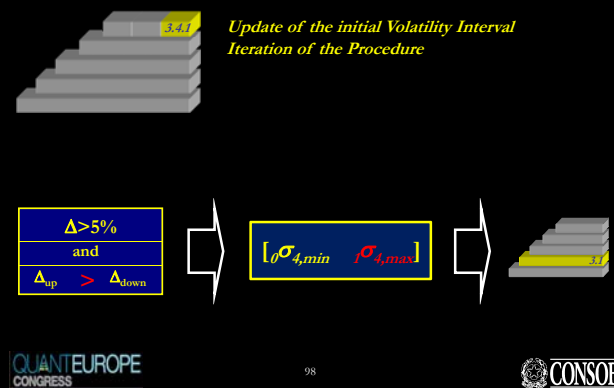
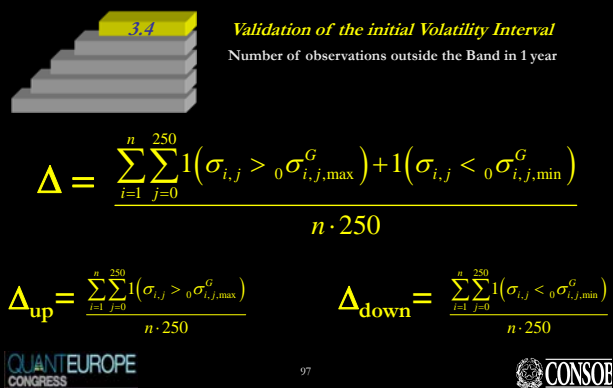
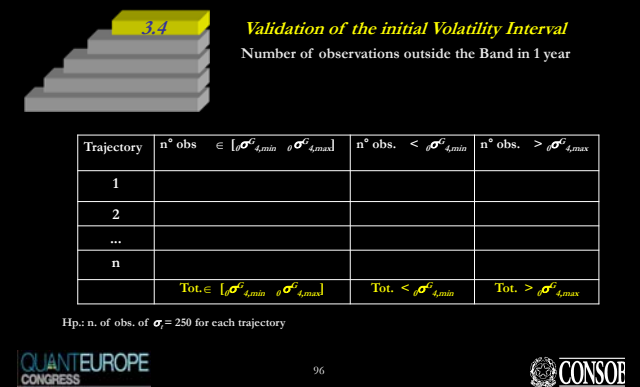
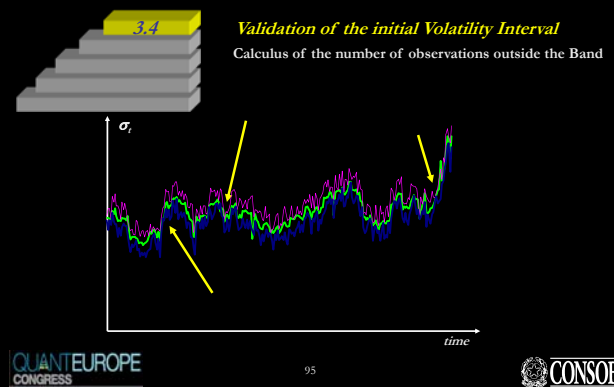
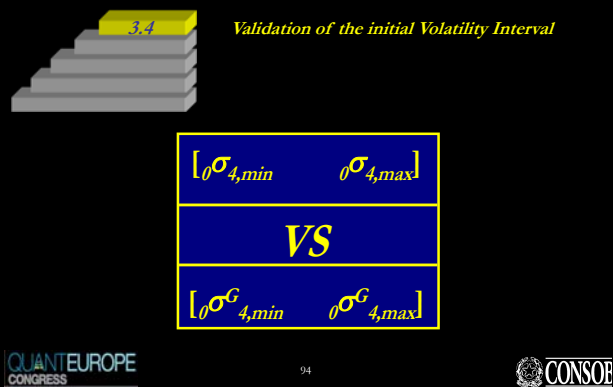
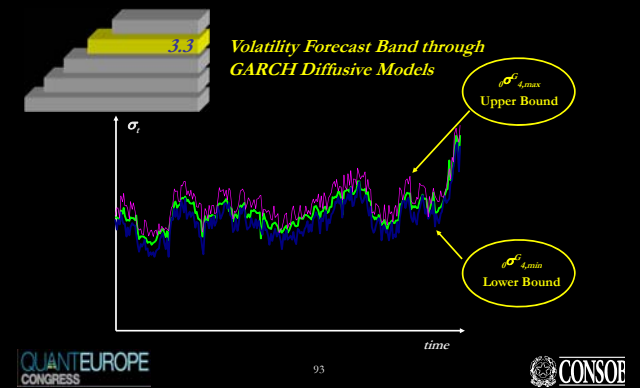
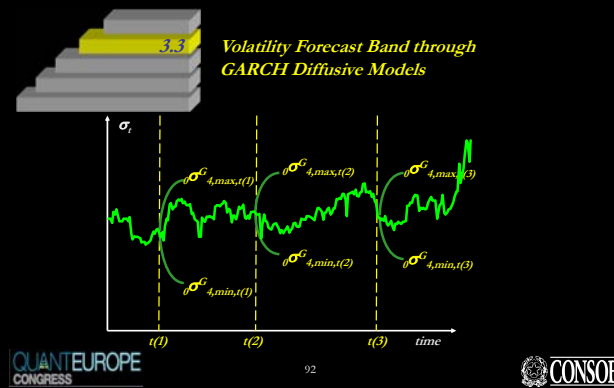
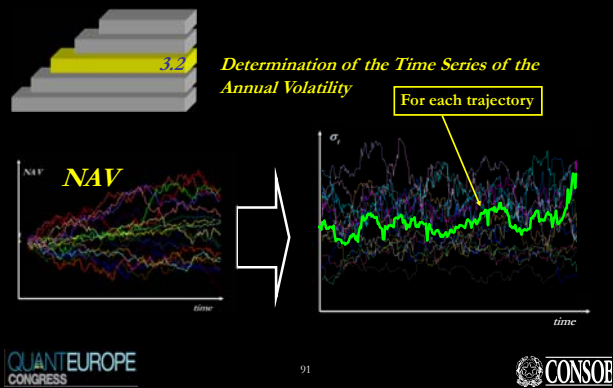
The Diffusion

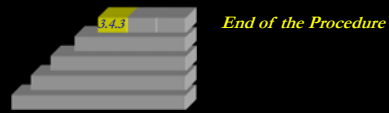


$[\sigma_{4,min}, \sigma_{4,max}]$

representativeness of volatility intervals





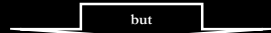


$$\Delta \leq 5\%$$

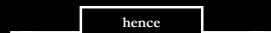
$$\begin{bmatrix} k\sigma_{4,min} & k\sigma_{4,max} \\ \sigma_{4,min} & \sigma_{4,max} \end{bmatrix} =$$

k = n. di iterations done

the iterative procedure guarantees that a fund belonging to a given risk class does not breach the GARCH adaptive band more than 5% of the days in 1 yr



migration risk is measured against fixed volatility intervals



output intervals are inherently prudential w.r.t. the 3 months migration rule

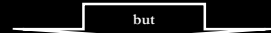
output volatility intervals

Risk Classes	Volatility Intervals <small>(annualized values)</small>	
	σ_{min}	σ_{max}
low	0.01%	0.49%
medium-low	0.50%	1.59%
medium	1.60%	3.99%
medium-high	4.00%	9.99%
high	10.00%	24.99%
very high	25.00%	above 25.00%

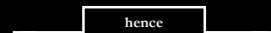
the iterative procedure guarantees that a fund belonging to a given risk class does not breach the GARCH adaptive band more than 5% of the days in 1 yr



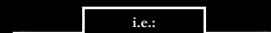
no more than 15 days over 250



migration risk is measured against fixed volatility intervals



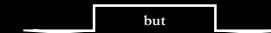
output intervals are inherently prudential w.r.t. the 3 months migration rule



output intervals are wide enough to avoid spurious migrations

Syllabus

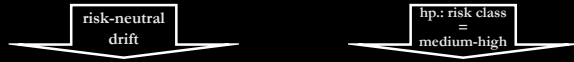
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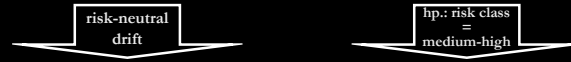
migration risk is measured against fixed volatility intervals

as confirmed by back-testing simulation

as confirmed by back-testing simulation



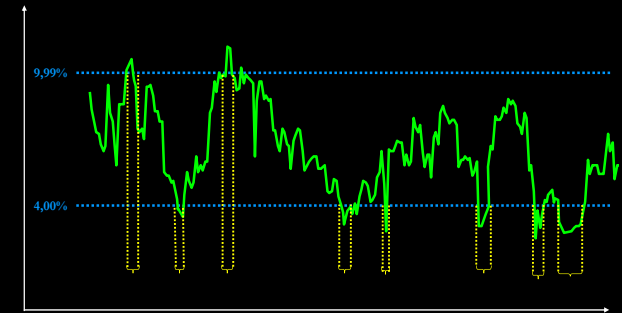
as confirmed by back-testing simulation



drawing diffusion from a symm. triang. distrib. bounded at $[\sigma_{min}, \sigma_{max}] = [4\%, 9.99\%]$ to simulate an hypothetic daily NAV and to calculate the corresponding realized volatility

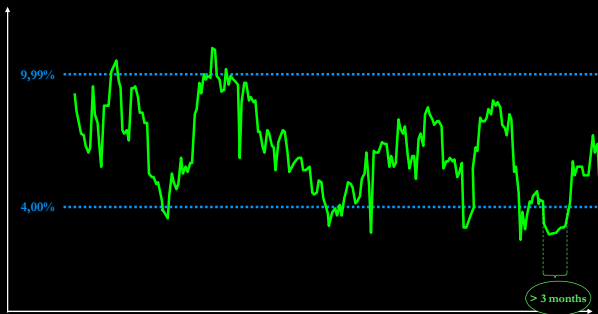
for each trajectory

as confirmed by back-testing simulation



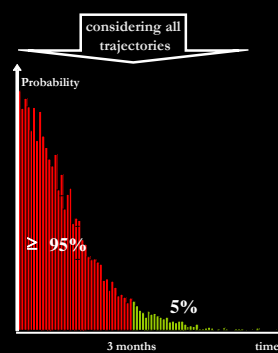
ALL THE BREACHES MARKED IN YELLOW LAST LESS THAN 3 MONTHS

as confirmed by back-testing simulation



THE OUTLIER MARKED IN GREEN LASTS MORE THAN 3 MONTHS

as confirmed by back-testing simulation



considering all trajectories

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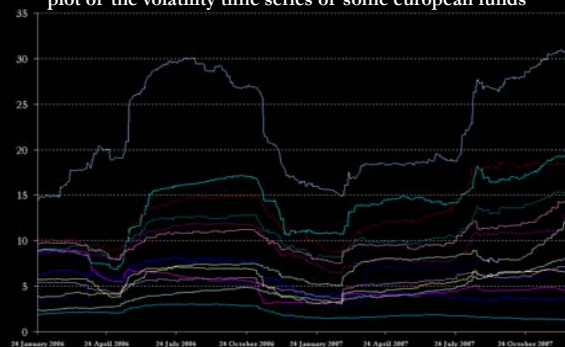
Empirical Evidence: Preliminary Informations

universe VS sample

Country	Total (A)	Selected (B)	Representativity (B/A)
Austria	17	13	76.5%
France	92	53	57.6%
Germany	63	45	71.4%
Ireland	2	1	50.0%
Italy	58	52	89.7%
Luxembourg	252	153	60.7%
Spain	224	130	58.0%
UK	8	7	87.5%
Total	716	454	63.4%

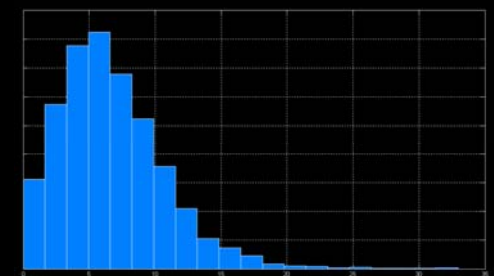
Empirical Evidence: Preliminary Informations

plot of the volatility time series of some european funds



Empirical Evidence: Preliminary Informations

volatility histogram for the sampled funds



initial distribution of the 454 funds between the 6 risk classes
(abs. values)

Country	Initial Risk Class as from 1st January 2006					Total
	1	2	3	4	5	
Austria	0	0	4	8	1	13
France	0	2	9	37	5	53
Germany	0	2	10	26	7	45
Ireland	0	1	0	0	0	1
Italy	1	11	11	28	1	52
Luxembourg	1	6	30	100	16	153
Spain	0	23	33	62	12	130
UK	0	0	0	5	2	7
Total	2	45	97	266	44	454

initial distribution of the 454 funds between the 6 risk classes
(perc. values)

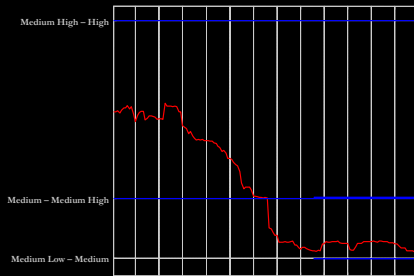
Country	Initial Risk Class as from 1st January 2006					Total
	1	2	3	4	5	
Austria	0.0%	0.0%	30.8%	61.5%	7.7%	100%
France	0.0%	3.8%	17.0%	69.8%	9.4%	100%
Germany	0.0%	4.4%	22.2%	57.8%	15.6%	100%
Ireland	0.0%	100.0%	0.0%	0.0%	0.0%	100%
Italy	1.9%	21.2%	21.2%	53.8%	1.9%	100%
Luxembourg	0.7%	3.9%	19.6%	65.4%	10.5%	100%
Spain	0.0%	17.7%	25.4%	47.7%	9.2%	100%
UK	0.0%	0.0%	0.0%	71.4%	28.6%	100%
Total	0.4%	9.9%	21.4%	58.6%	9.7%	100%

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MIGRATION



— Risk Class as from the Prospectus
— Risk effectively taken

n. of migrations between different risk classes: 01.01.2006 – 12.31.2007
(abs. values)

Country	Number of Migrations over the period January 2006 - December 2007						Total
	0	1	2	3	4	5	
Austria	2	6	2	3	0	0	13
France	20	13	8	11	1	0	53
Germany	18	6	13	8	0	0	45
Ireland	0	1	0	0	0	0	1
Italy	15	12	17	8	0	0	52
Luxembourg	63	28	34	23	4	1	153
Spain	44	30	31	21	4	0	130
UK	1	3	1	2	0	0	7
Total	163	99	106	76	9	1	454

migrations per Country: 01.01.2006 – 12.31.2007
(perc. values)

Country	0	1	2	3	4	5	Total
Austria	15.4%	46.2%	15.4%	23.1%	0.0%	0.0%	100%
France	37.7%	24.5%	15.1%	20.8%	1.9%	0.0%	100%
Germany	40.0%	13.3%	28.9%	17.8%	0.0%	0.0%	100%
Ireland	0.0%	100.0%	0.0%	0.0%	0.0%	0.0%	100%
Italy	28.8%	23.1%	32.7%	15.4%	0.0%	0.0%	100%
Luxembourg	41.2%	18.3%	22.2%	15.0%	2.6%	0.7%	100%
Spain	33.8%	23.1%	23.8%	16.2%	3.1%	0.0%	100%
UK	14.3%	42.9%	14.3%	28.6%	0.0%	0.0%	100%

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Conclusions

- ✓ REGULATORY ISSUE:
measurement and representation of mutual funds risk profile
- ✓ ADEQUACY REQUIREMENT OF THE APPROACH:
ability to pursue full risk disclosure consistently with the “standard” activity of fund managers
- ✓ DIFFUSIVE GARCH:
definition of adaptive volatility prediction bands
- ✓ EMPIRICAL EVIDENCE:
the phenomenon of the migration interests more the funds belonging to the riskiest classes
- ✓ CLOSING RECOMMENDATION:
exploring other fields of application, especially to move faster towards a really levelled playing field

QUANTEUROPE CONGRESS

CONSOB

VOLATILITY METRICS TO ASSESS RELATIVE RISK IN THE
QUANTITATIVE PORTFOLIO MANAGEMENT OF MUTUAL FUNDS:
A REGULATORY APPROACH BASED ON DIFFUSIVE GARCH

LONDON, NOVEMBER 6TH 2008

MARCELLO MINENNA
GIOVANNA MARIA BOI